Int. J. Production Economics 181 (2016) 98–106

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Optimally configuring a two-stage serial line supply chain under the guaranteed service model

N. Grace Hua [a], Sean P. Willems [b,*]

[a] Microsoft Corporation, Redmond, WA 98052, USA United States
[b] Haslam College of Business, The University of Tennessee, Knoxville, TN 37996, United States

ARTICLE INFO

Article history:
Received 30 September 2014
Received in revised form
27 April 2016
Accepted 6 June 2016
Available online 25 June 2016

Keywords:
Supply chain sourcing and design
Multiechelon inventory system
Supply chain configuration

ABSTRACT

Design-and-development buyers sourcing new supply chains and strategic sourcing analysts making outsourcing decisions for existing supply chains are in different organizations but share a common problem. Both determine whether an existing part should be replaced. Relative to the existing part, new candidates could be cheaper with longer leadtime, or more expensive with shorter leadtime. Furthermore, a longer leadtime part could be buffered with inventory and this could be cheaper than paying more for a shorter leadtime part.

To derive analytical insights into the nature of this problem, we restrict our scope to a two-stage serial line supply chain. This restriction is consistent with sourcing analysts that consider sourcing a single part from different vendors and different transportation alternatives. The resulting two-stage supply chain configuration model jointly determines the chosen option and inventory stocking level at each stage to minimize cost of goods sold, pipeline stock cost and safety stock cost.

We prove it is preferable to synchronize the supply chain by employing the same type of option, either low cost long leadtime or high cost short leadtime, at both stages. We prove that the selection threshold for high cost short lead time options is lowest at just the downstream stage, highest for just the upstream stage, and between these extremes if such a candidate is selected for both stages. If a part's cost-time relationship follows a functional form, we establish conditions when it is optimal to choose the lowest cost, longest leadtime, option available.

1. Introduction

This problem was first encountered when observing supply chain design and development (D&D) buyers source new product supply chains at Eastman Kodak. D&D buyers source new product supply chains after the product has cleared the approval gates of the new product development process. As documented in Graves and Willems (2005), the buyer would import the previous generation's bill of materials and use it as a starting point for the new supply chain's design. While major components (like an imager or flash assembly) would be changed every generation, even these items would begin with last year's information. In a bill of materials that comprises thousands of items, less than a hundred would surely change and the rest would only change if a business case justified the change.

In a different context, this problem was next encountered in the strategic sourcing departments at companies including Cisco, Hewlett-Packard, Moen, Procter & Gamble, and Stanley Black & Decker. These companies have significant global footprints comprised of internal and external manufacturing sites. The strategic sourcing department is a cost center that supports the lines of business. Strategic sourcing does not architect new supply chains. They are arbiters of fact. With the exception of parts shared across business units, which falls under commodity management, strategic sourcing performs a rigorous landed cost analysis of alternatives but they are not the subject matter experts creating the alternatives. They take the set of alternatives provided by the business and evaluate them on financial grounds. As such, strategic sourcing does not have the power to radically architect the entire supply chain. Instead, each week it was being tasked with evaluating whether a set of parts should be moved from one site to another. The resulting analysis was performed on a per-part basis.

In both of these problem settings, we observed practitioners view sourcing within a framework where the current part was only replaced if a new choice lowered total supply chain cost. Total supply chain cost was defined as the sum of cost of goods sold (COGS, i.e. unit cost times volume) and inventory cost (pipeline

* Corresponding author.
E-mail addresses: nih@microsoft.com (N.G. Hua), swillems@utk.edu (S.P. Willems).

http://dx.doi.org/10.1016/j.ijpe.2016.06.007
0925-5273/© 2016 Published by Elsevier B.V.

and safety stock). These three cost components are minimally sufficient to capture the relationship between cost and time in the supply chain. Practitioners know inventory can buffer long part leadtimes but they also know there is a point at which the extra inventory cost offsets the unit cost savings associated with the cheaper part.

As a specific example, in 2011 we worked with a company's strategic sourcing group that received over 200 part outsourcing requests per month; they were referred to as outsourcing requests because the parts were currently produced in North America and the alternative supply locations were international sites that might or might not be company owned. For each part, a qualitative assessment was made about whether it was a feasible candidate to consider for outsourcing. This included evaluating technological, strategic and intellectual-property dimensions. If the part passed these hurdles, the strategic sourcing group performed a landed-cost analysis considering the purchasing, transportation, and inventory costs to deliver the part to its North American manufacturing site. Incumbent parts ranged in cost from 50 cents to several hundred dollars, and leadtimes ranged from a few days to several months. The parts included in the analysis were brought forward in a piecemeal fashion as different product subassemblies had sourcing reviews. It was not uncommon for a single part from a product comprised of thousands of parts to be considered one week and then three weeks later a different single part from the product would be brought forward.

This paper analytically characterizes when an existing option in the supply chain should be replaced by a candidate option. The restriction to a two-stage serial line network is more stylized than the problem faced by the D&D buyers but it exactly matches the problem faced by the strategic sourcing teams. For example, for a single part the part being considered is one stage while the other stage is the transportation process; or a more high-level analysis can be constructed where a single stage is the cost to deliver the part on site and the remainder of the supply chain is approximated by a single stage. Furthermore, reducing scope to two stages allows us to develop analytical results that provide intuition to guide sourcing decisions.

The paper is structured as follows. Section 2 contains a literature review. Section 3 presents notation and modeling assumptions, introducing the supply chain configuration problem in the context of a two-stage serial line supply chain. Section 4 frames the problem in the context that supply chain practitioners consider the problem: in a setting where incumbent options that source the existing supply chain are considered against candidate options. Section 5 abstracts the notion of an option to be specified as a functional relationship between cost and leadtime. Section 6 concludes with a summary and future research directions.

2. Literature review

There are several research streams addressing the tradeoff of cost and leadtime in the supply chain. The first is in the traditional inventory literature. Gross and Soriano (1969) note that faster shipping can reduce the required pipeline and safety stock inventory. They show that the higher the service level is, the greater the potential to save from reducing leadtime. Bertazzi et al. (2000) consider the problem of shipping several products from an origin to a destination when a discrete set of shipping frequencies are available and the objective is to minimize the sum of inventory and transportation cost. Cetinkaya and Lee (2000) determine the optimal replenishment and dispatch policy in a vendor-managed inventory system by minimizing the total inventory, procurement, transportation, and waiting costs.

There has been significant activity in the areas of dynamic leadtime management and dual sourcing. This research is more operational in nature and focused on both optimal order allocation between sources at a single stage and the optimal inventory policy; we refer the reader to Minner (2003), Thomas and Tyworth (2006) and Boute and Van Mieghem (2011) for their thorough literature reviews. Various supply options such as dual supply modes (Anupindi and Akella, 1993, Babich et al., 2007, Tomlin, 2009), expediting and emergency orders (Lawson and Porteus, 2000, Huggins and Olsen, 2003, Muharremoglu and Tsitsiklis, 2003, Jain et al., 2010), are utilized in order to make the leadtime flexible enough to satisfy demand fluctuation.

To the best of the authors' knowledge, there does not exist literature prior to Graves and Willems (2005) that frames supply chain configuration in terms of incumbent and candidate options differing in cost and leadtime.

The modeling framework in our paper applies Graves and Willems (2005) with a reduction in scope from a spanning tree to a two-stage serial line supply chain. A two-stage serial line is a reasonable approximation for a single-part outsourcing analysis consisting of a part and its transportation process or for one part isolated from an aggregation of the remaining supply chain. Furthermore, this scope reduction allows us to analytically determine structural properties and gain insights into the dynamics of the problem.

3. Model assumptions and formulation

The following assumptions underlie the supply chain configuration problem in a two-stage serial line supply chain. The supply chain is modeled as a network, where stages denote functional performance requirements that have to be met, for example, the production of a part, or the requirement that a certain part needs to be transported to a warehouse. The arcs denote the precedence relationship between stages. The upstream stage is denoted as stage 2 while the downstream stage is denoted as stage 1. Different sourcing options in each stage may result in a different optimal safety stock inventory holding strategy, which adds to the complexity of the problem.

3.1. Option definition

Each option at a stage is defined as a cost and leadtime pairing. Cost includes the direct material and labor costs associated with the option, and leadtime is the time required to transform the stage's input to output. If the stage represents a transportation function, one option could represent air transport (with a high cost and short leadtime) while another option represents truck transport (with a low cost and long leadtime). We assume the product's design has already been decided. Therefore, the network's structure is fixed and the outstanding questions are which option to select, and how much inventory to hold, at each stage in the supply chain.

3.2. Guaranteed service model for inventory placement

Our assumptions regarding the demand and fulfillment process at each stage are based on the guaranteed service model of safety stock optimization, originally defined by Simpson (1958). Each stage quotes a nonnegative outgoing service time, which is the amount of time between receiving an order from a downstream stage and satisfying the order. This differs from leadtime which is the time to convert the stage's input to an output available to satisfy demand. Service time is guaranteed in the sense that as long as the demand is less than a specified upper bound, the order is committed to be delivered within the outgoing service time. Without loss of generality, we assume that stage 1 must provide immediate delivery for external demand, and that stage 2 has immediate access to raw materials; this is equivalent to assuming stage 1's outgoing service time is zero and the maximum outgoing service time quoted to stage 2 is zero.

We assume external demand occurs only at stage 1 and one unit from stage 2 is required to satisfy one unit of demand at stage 1. Furthermore, we assume the demand process at stage 1 is stationary with average per period demand μ and standard deviation σ. Demand at a stage is bounded by the function $D(\tau)$ which specifies an upper bound on demand for any interval of length τ. Such a demand bound does not mean that demand can never exceed the bound but it does mean that the safety stock in the system is only designed to satisfy the demand within the bound. Assuming a service factor of k, coupled with the fact that we assume one from stage 2 is required to satisfy one unit do demand at stage 1, $D(\tau)$ can be defined as $\mu\tau+k\sigma\tau^{1/2}$. This functional form is well documented to be the method the guaranteed service model implements $D(\tau)$ in practice, although it is not always assumed so directly at the start. Specific examples are provided in Graves and Willems (2008) and Bossert and Willems (2007), while Willems (2008) presents 38 supply chains from 29 companies that all employ this functional form for specifying demand.

The power of the guaranteed service model is the fact that it does not try to model what happens if demand exceeds the demand bound; it assumes the service time is guaranteed so appropriate countermeasures are taken when demand exceeds the bound. While this is admittedly a strong assumption, Graves and Willems (2003) show it is no stronger than the assumption made by other literature that assumes the system will behave like a queueing system that assume backlogging and wait until the backlogged unit arrives. In effect, these two reactions represent the extremes of what would happen in reality.

3.3. Optimization model

We denote the supply chain configuration optimization problem as **P**. The decision variables in problem **P** are the option selected at each stage and stage 2's outgoing service time.

$$\mathbf{P} \quad \min \beta(C_1 + C_2)\mu + \tfrac{1}{2}\alpha C_2 T_2 \mu + \alpha\left(\tfrac{1}{2}C_1 + C_2\right)T_1\mu + \alpha C_2 k\sigma\sqrt{\tau_2} + \alpha(C_1 + C_2) k\sigma\sqrt{\tau_1}$$

$$s.\,t. \quad \sum_{j=1}^{O_i} t_{ij}y_{ij} - T_i = 0 \qquad \text{for } i = 1, 2, \tag{1}$$

$$\sum_{j=1}^{O_i} c_{ij}y_{ij} - C_i = 0 \qquad \text{for } i = 1, 2, \tag{2}$$

$$\tau_2 = T_2 - S_2 \tag{3}$$

$$\tau_1 = S_2 + T_1 \tag{4}$$

$$\tau_i \ge 0 \qquad \text{for } i = 1, 2, \tag{5}$$

$$S_2 \ge 0 \tag{6}$$

$$\sum_{j=1}^{O_i} y_{ij} = 1 \qquad \text{for } i = 1, 2, \tag{7}$$

$$y_{ij} \in \{0, 1\} \qquad \text{for } i = 1, 2, 1 \le j \le O_i \tag{8}$$

where

O_i = number of available options at stage i.
c_{ij} = direct cost of option j at stage i.
t_{ij} = leadtime of option j at stage i.
y_{ij} = indicator variable which equals 1 if stage i's jth option is selected and 0 otherwise.
C_i = direct cost of option selected at stage i.
T_i = leadtime of option selected at stage i.
S_i = outgoing service time at stage i.
τ_i = net replenishment time at stage i.
α = holding cost rate.
β = scalar converting model's time period into company's time interval of interest.
σ = standard deviation of demand per period.
μ = mean demand per period.

The first term of **P**'s objective function comprises COGS while the second and third terms correspond to pipeline stock cost and the final two terms capture safety stock cost. To simplify the model formulation, we assume that both stages have the same holding cost rate. We choose α and β to reflect the same interval of time. For example, if the company operates for 250 business days a year and the model's time and demand inputs are expressed as daily units, then to calculate annual supply chain configuration cost requires β to equal 250 while a 45% annual holding cost rate would require α to equal 0.45.

Constraints (1) and (2) define the cost and leadtime for each stage, as they depend on the stage's chosen option. Constraints (3) and (4) define the net replenishment time for each stage; the constraints are different because we assume that stage 2 can get its raw materials immediately and that stage 1 holds sufficient inventory to meet external demand immediately from stock. Constraint (5) requires the net replenishment times to be nonnegative and (6) requires the outgoing service time from stage 2 to be nonnegative. Constraints (7) and (8) enforce the sole sourcing of options.

4. Candidate versus incumbent options, under general cost-time pairings

In this section, we require no functional relationship between an option's cost and leadtime, and consider candidate options at each individual stage as well as at both stages. In practice, there is often no discernible relationship between an option's cost and leadtime, or between alternative options. As shown in Graves and Willems (2005), while one supplier might offer a menu of prices that relate to different delivery time commitments, it was often the case that different suppliers would quote vastly different cost-leadtime pairings based on where the part fit in their strategic and competitive priorities.

While every option is a cost-time pairing, companies are not looking at all options the same way. In particular the options that sourced the existing supply chain are viewed as incumbent options and the set of all other options are candidate options. In terms of notation, we will denote the incumbent option as the first option at stage i, i.e. (c_{i1}, t_{i1}), and all candidate options as (c_{ij}, t_{ij}) where $j > 1$. The incumbent network is the two-stage supply chain configuration that employs each stage's incumbent option.

For either stage, Fig. 1 presents a quadrant-based view of candidate options relative to the stage's incumbent option. The X-axis measures stage leadtime and the Y-axis measures stage cost. With the incumbent option represented by the origin, a candidate option's placement on the graph is found by the point $(c_{ij}-c_{i1}, t_{ij}-t_{i1})$. If we only consider changing the option at one stage, it is obvious that an incumbent option would never be replaced by a candidate option from quadrant I because this candidate option would have a higher leadtime and a higher cost. Conversely, a candidate option in quadrant III would always be selected because it dominates the incumbent option by having a shorter leadtime and lower cost. The interesting cases we need to focus on are candidate options in quadrants II and IV, which will be called Type II and Type IV candidate options. For these options, one dimension improves but the other worsens.

To formalize the notion of a candidate option versus the incumbent option, we define ΔC as the absolute value of the cost difference between the incumbent and a candidate option and ΔT as the absolute value of the leadtime difference. We will see that the ratio $\Delta C/\Delta T = |c_{ij} - c_{i1}|/|t_{ij} - t_{i1}|$ will dictate whether or not a candidate option lowers total supply chain configuration cost versus employing the incumbent option.

4.1. Decision rule for selecting the candidate option at stage 2

In this subsection, we only consider candidate options at stage 2; the option at stage 1 is fixed to be the incumbent option. The resulting mathematical program is a constrained version of **P** that we denote as $\mathbf{P^2}$. $\mathbf{P^2}$ can be stated compactly as:

$$\mathbf{P^2} \quad \min \beta(C_2 + c_{11})\mu + \frac{1}{2}\alpha C_2 T_2 \mu + \frac{1}{2}\alpha(2C_2 + c_{11})t_{11}\mu + \alpha C_2 k\sigma\sqrt{\tau_2} + \alpha(c_{11} + C_2)k\sigma\sqrt{\tau_1}$$

$$s.\ t. \quad \sum_{j=1}^{O_2} t_{2j}y_{2j} - T_2 = 0;\ \sum_{j=1}^{O_2} c_{2j}y_{2j} - C_2 = 0;\ \tau_2 = T_2 - S_2;\ \tau_1 = S_2 + t_{11};\ \tau_1 \geq 0;$$

$$\tau_2 \geq 0;\ S_2 \geq 0;\ \textstyle\sum_{j=1}^{O_2} y_{2j} = 1;\ y_{2j} \in \{0, 1\} \quad for\ \ 1 \leq j \leq O_2.$$

We let $h(c_{21}, t_{21})$ denote the objective function value of $\mathbf{P^2}$ when the incumbent option at stage 2 is employed; i.e., when $y_{21}=1$ and $y_{2j}=0$ for $j>1$. $h(c_{21}, t_{21})$ is the objective function value of the incumbent network, and it establishes the benchmark to evaluate when candidate options at stage 2 lower supply chain configuration cost relative to the incumbent option. The optimal inventory deployment in the incumbent network either holds safety stock inventory only at stage 1 or at both stages. We let $h_1(c_{21}, t_{21})$ denote the first scenario, where $\tau_2 = t_{21}$, and $h_2(c_{21}, t_{21})$ the second, where $\tau_2 = 0$. Thus, $h(c_{21}, t_{21}) = \min(h_1(c_{21}, t_{21}), h_2(c_{21}, t_{21}))$. Neither $h_1(c_{21}, t_{21})$ nor $h_2(c_{21}, t_{21})$ is a concave function but Lemma 1 proves that they are both quasi-concave, thus $h(c_{21}, t_{21})$ is also quasi-concave.

Lemma 1. *Both $h_1(c_{21}, t_{21})$ and $h_2(c_{21}, t_{21})$ are quasi-concave functions, therefore $h(c_{21}, t_{21}) = \min(h_1(c_{21}, t_{21}), h_2(c_{21}, t_{21}))$ is also a quasi-concave function.*

In order to use Lemma 1 to establish sufficient conditions for the selection of candidate options over the incumbent option, we need to define two critical ratios. Let.

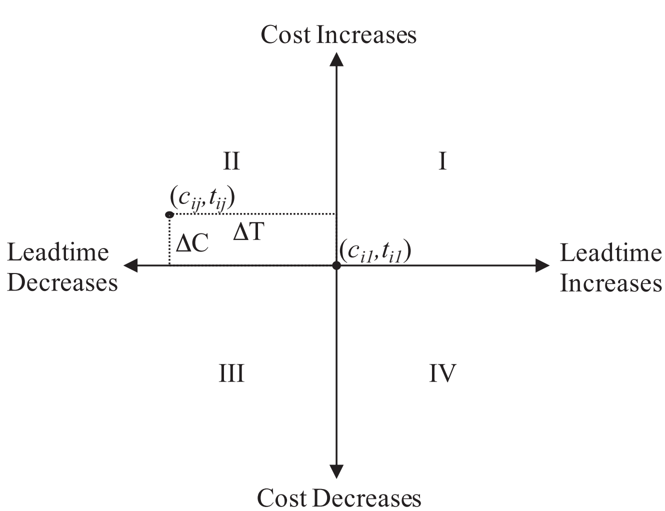

Fig. 1. Quadrant-based view of candidate options versus incumbent option.

$$R_{21} = \left\{\alpha(c_{11} + c_{21})k\sigma\frac{1}{2\sqrt{t_{11}+t_{21}}} + \frac{1}{2}\alpha c_{21}\mu\right\} \Big/ \left\{\alpha k\sigma\sqrt{t_{11} + t_{21}} + \frac{1}{2}\alpha t_{21}\mu + \alpha t_{11}\mu + \beta\mu\right\}$$

and $R_{22} = \left\{\alpha c_{21}k\sigma\frac{1}{2\sqrt{t_{21}}} + \frac{1}{2}\alpha c_{21}\mu\right\} \Big/ \left\{\alpha k\sigma\sqrt{t_{21}} + \alpha k\sigma\sqrt{t_{11}} + \frac{1}{2}\alpha t_{21}\mu + \alpha t_{11}\mu + \beta\mu\right\}.$

For R_{ij}, the subscript i represents the stage ($i=1,2$) while j differentiates the scenarios where a decoupling safety stock is not held ($j=1$) and held ($j=2$) at stage 2. With these ratios defined, we have the following two theorems.

Theorem 1. *When considering a Type II candidate option (c_{2j},t_{2j}) at stage 2, if it is optimal for the incumbent network to only hold safety stock at stage 1, select the candidate option if $\Delta C/\Delta T < R_{21}$; if it is optimal for both stages to hold safety stock in the incumbent network, select the candidate option if $\Delta C/\Delta T < R_{22}$.*

Theorem 2. *When considering a Type IV candidate option (c_{2j},t_{2j}) at stage 2, if it is optimal for incumbent network to only hold safety stock at stage 1, select the candidate option if $\Delta C/\Delta T > R_{21}$; if it is optimal for both stages to hold safety stock in the incumbent network, select the candidate option if $\Delta C/\Delta T > R_{22}$.*

To provide the intuition for Theorem 1 and 2, Fig. 2 shows the contours of $h(c_{21},t_{21})$ and the function $1 - \frac{c_{21}}{c_{11}+c_{21}}\sqrt{\frac{t_{21}}{t_{11}+t_{21}}} - \sqrt{\frac{t_{11}}{t_{11}+t_{21}}} = 0$, which marks the boundary of whether or not it is optimal to hold safety stock at stage 2 in the incumbent network. In this example, the holding cost rate is 45% and for the purposes of calculating COGS there are 250 days in the horizon. Average demand is 100 per day and standard deviation of demand is 80. The safety factor is 3. These parameters are chosen to be consistent with the example in Graves and Willems (2005) and they are further validated as being reasonable by Callioni et al. (2005). The incumbent option at stage 1 has $c_{11}=70$ and $t_{11}=30$.

Any contour line above the safety stock boundary is of the function $h_1(c_{21},t_{21})$, since in this region $h_1(c_{21},t_{21}) < h_2(c_{21},t_{21})$. Therefore, the optimal safety stock policy for incumbent options in this region holds safety stock only at stage 1. The contour lines below the boundary are of the function $h_2(c_{21},t_{21})$. The fact that the optimal cost function is different on each side of the boundary explains why there are two thresholds when considering candidate options at stage 2.

In this specific example, point A represents stage 2's incumbent option ($c_{21}=58$ and $t_{21}=82$) and with that option safety stock is held at stage 2. It is clear from the graph that there are Type II and Type IV candidate options that have a total supply chain configuration cost contour below the contour associated with the incumbent option. In fact, the shaded areas of quadrant II and IV denote the set of candidate options in these quadrants that meet the sufficient conditions established in Theorems 1 and 2; this set of candidate options has a lower total supply chain configuration cost than the incumbent option. And it is also clear from Fig. 2 that for a Type II candidate option to be selected, we need its ratio $\Delta C/\Delta T$ to be sufficiently small, while for a Type IV candidate, we need $\Delta C/\Delta T$ to be sufficiently large.

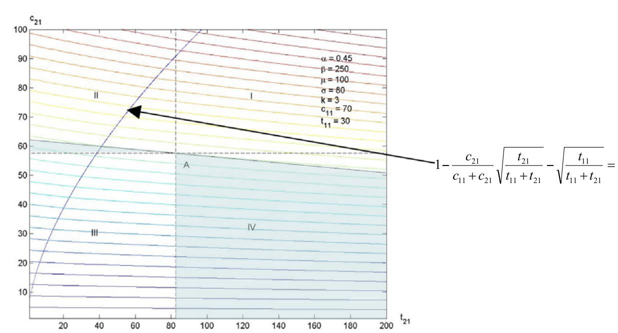

Fig. 2. *$h(c_{21}, t_{21})$* contour map.

Corollary 1. *R_{21} is an increasing function of c_{11}, c_{21}, and α, and a decreasing function of β, t_{11}, and t_{21}. R_{22} is an increasing function of c_{21} and α, and a decreasing function of β, t_{11}, and t_{21}.*

Theorem 1 and 2 jointly show that $[0, R_{2j}]$ is the range for a Type II candidate option to be selected, while $[R_{2j}, \infty)$ is the range for the selection of a Type IV candidate option. Therefore, when R_{2j} is increased, the feasible region for Type II candidate options that lower supply chain configuration cost versus the incumbent option increases while the feasible region for Type IV candidate options decreases correspondingly. Combined with Corollary 1, we first draw three analytically-based conclusions and then we share the primary takeaway gleaned from the strategic sourcing teams.

First, the relative importance of COGS versus safety and pipeline stock cost defines the extent to which Type II or Type IV candidate options can reduce supply chain configuration cost. Besides the cost and leadtime parameters of the options selected in the supply chain, the holding cost rate α and the number of periods in the horizon β act to weight the three components of supply chain configuration cost. Since R_{21} and R_{22} are increasing functions of α and decreasing functions of β, the benefit of choosing a Type II candidate option increases, and the benefit for choosing a Type IV candidate option decreases, as safety stock and pipeline stock cost become more significant relative to COGS in the calculation of supply chain configuration cost.

Second, the presence of a high cost short leadtime incumbent at stage 1 increases the set of Type II candidate options that lower supply chain configuration cost versus the incumbent option at stage 2. Conversely, a low cost long leadtime incumbent at stage 1 increases the set of Type IV candidate options that lower supply chain configuration cost versus the incumbent option at stage 2. Informally speaking, this phenomenon can be thought of as synchronizing the supply chain; the intuition is that if stage 1 employs a low cost long leadtime option then using a high cost short leadtime option at stage 2 most likely will not reduce the supply chain's cost because the leadtime saved at stage 2 is offset by the long leadtime at stage 1. But if both stages are using high cost short leadtime options, the supply chain as a whole is more responsive, with the savings from the safety stock and pipeline stock inventory outweighing the increased COGS and thus lowering supply chain configuration cost.

The last interesting result from Corollary 1 is that R_{22}, which is the threshold for the scenario where it is optimal for a decoupling inventory at stage 2 in the incumbent network, is not a function of c_{11}. When the inventory placement in the system is decoupled, c_{11} only has a local impact on the three components of the supply chain configuration cost and does not interact with either the direct leadtime or the cost at stage 2, and thus does not play a role in determining R_{22}.

The primary takeaway practitioners took from this analysis was that the hurdle to select a Type II candidate option was high. That leadtime had to decrease significantly to warrant even a small increase in cost. In contrast, on a relative basis, small decreases in cost could justify large increases in leadtime. The strategic sourcing team used this knowledge to inform the supply chain teams what parts could be good candidates to outsource since it is the supply chain teams that have a sense about what alternative choices might be available.

4.2. Decision rule for selecting the candidate option at stage 1

This section fixes the incumbent option at stage 2 and considers candidate options at stage 1. Similar to Section 4.1, we can define $\mathbf{P^1}$ as the mathematical program that constrains stage 2 to use the incumbent option and optimizes the option selected at stage 1. We let $g(c_{11}, t_{11}) = \min\{g_1(c_{11}, t_{11}), g_2(c_{11}, t_{11})\}$ where $g(c_{11}, t_{11})$ is the minimum supply chain configuration cost in the incumbent network, $g_1(c_{11}, t_{11})$ and $g_2(c_{11}, t_{11})$ represent the scenarios where only stage 1 holds safety stock inventory and both stages hold safety stock inventory. As in Section 4.1, we can prove that $g(c_{11}, t_{11})$ is quasi-concave. We maintain our definitions of ΔC and ΔT and introduce two new critical ratios.

$$R_{11} = \left\{\alpha(c_{11}+c_{21})k\sigma\frac{1}{2\sqrt{t_{11}+t_{21}}} + \frac{1}{2}\alpha(2c_{21}+c_{11})\mu\right\} \Big/ \left\{\alpha k\sigma\sqrt{t_{11}+t_{21}} + \frac{1}{2}\alpha t_{11}\mu + \beta\mu\right\}$$

and.

$$R_{12} = \left\{\alpha(c_{11}+c_{21})k\sigma\frac{1}{2\sqrt{t_{11}}} + \frac{1}{2}\alpha(2c_{21}+c_{11})\mu\right\} \Big/ \left\{\alpha k\sigma\sqrt{t_{11}} + \frac{1}{2}\alpha t_{11}\mu + \beta\mu\right\}.$$

With this notation, we can now provide analytical conditions for the acceptance of candidate options at stage 1.

Theorem 3. *When considering a Type II candidate option (c_{1j},t_{1j}) at stage 1, if it is optimal for the incumbent network to hold safety stock at stage 1, select the candidate option if $\Delta C/\Delta T < R_{11}$; if it is optimal for both stages to hold safety stock, select the candidate option if $\Delta C/\Delta T < R_{12}$.*

Theorem 4. *When considering a Type IV candidate option (c_{1j},t_{1j}) at stage 1, if it is optimal for the incumbent network to hold safety stock at stage 1, select the candidate option if $\Delta C/\Delta T > R_{11}$; if it is optimal for both stages to hold safety stock, select the candidate option if $\Delta C/\Delta T > R_{12}$.*

Fig. 3 provides intuition for Theorems 3 and 4.

Now the contour above the curve $1 - \frac{c_{21}}{c_{11}+c_{21}}\sqrt{\frac{t_{21}}{t_{11}+t_{21}}} - \sqrt{\frac{t_{11}}{t_{11}+t_{21}}} = 0$ is of the function $g_2(c_{11}, t_{11})$, and the options above the curve require both stages to hold safety stock inventory. Point B is in the region below the curve, corresponding to the scenario where only stage 1 holds safety stock inventory in the incumbent network. By Theorems 3 and 4, all candidate options lying in the shaded area in quadrant II and IV lower supply chain configuration cost compared to the incumbent option.

Corollary 2. *R_{11} is an increasing function of c_{11}, c_{21}, α, and a decreasing function of β, t_{11}, t_{21}. R_{12} is an increasing function of c_{11}, c_{21}, α, and a decreasing function of β and t_{11}.*

The insights that Theorems 3 and 4 as well as Corollary 2 provide for stage 1 are similar to those of stage 2. It is still true that when the inventory cost increases relative to total supply chain configuration cost, the attractiveness of choosing a Type II candidate option at stage 1 is increased.

The concept of supply chain synchronization is again present. By Corollary 2, when the incumbent option at stage 2 employs a higher cost and shorter leadtime option, the set of Type II candidate options at stage 1 that lower supply chain configuration cost versus the incumbent option increases while the set decreases for Type IV candidate options.

The last noteworthy point is that R_{12}, the threshold for the scenario where the incumbent network holds safety stock inventory at stage 2, is a function of c_{21} but not t_{21}. Again, this is due to the decoupling nature of the safety stock in the system. The leadtime at

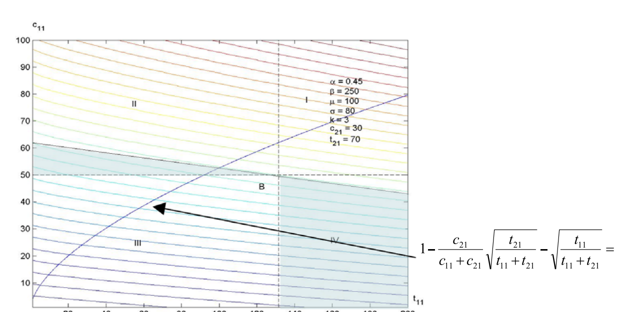

Fig. 3. $g(c_{11}, t_{11})$ contour map.

stage 2 does not have an impact on stage 1 decisions. However, the cost at stage 2 affects both safety stock and pipeline stock cost at stage 1, and therefore has an impact on the stage 1 decision.

Theorem 5 compares the selection thresholds when candidates are only considered at either stage 1 or stage 2.

Theorem 5. *Under the same system parameter settings, the threshold to select a higher cost shorter leadtime candidate option at the downstream stage is lower than the upstream stage; i.e.,* $R_{21} < R_{11}$ *and* $R_{22} < R_{12}$.

Increasing R_{ij} increases the feasible set of Type II candidate options that lower supply chain configuration cost relative to the incumbent option while decreasing the set of Type IV candidate options. Therefore, $R_{21} < R_{11}$ shows that if stage 2 does not hold safety stock under the incumbent network, the threshold for stage 2 to select the Type II candidate option is less than the threshold for stage 1; i.e., the feasible region for Type II candidate options is larger at stage 1 than stage 2. By $R_{22} < R_{12}$, it is shown that the same conclusion holds if under the incumbent network stage 2 holds safety stock inventory.

Choosing a Type II candidate option at either stage definitely increases COGS but choosing the Type II candidate at the upstream stage also definitely increases the pipeline stock cost at the downstream stage. To make financial sense, this increase has to be more than offset by savings from safety stock cost and possibly pipeline stock at the upstream stage. Thus the fewer successors a stage has, the less cost the Type II candidate option incurs downstream, and the easier it is to offset these increased cost components.

4.3. Considering candidate options at both stages

We now consider the scenario where both stages evaluate candidate options. Allowing candidates at both stages significantly increases the problem's complexity. In order to capture the problem richness of varying the options at both stages while still allowing analytical insights, we simplify **P** by assuming that the selected options at stage 1 and 2 differ by a constant multiplier. Mathematically speaking, we represent the problem as **P'**.

$$\mathbf{P'}\quad \min \beta(C_1 + C_2)\mu + \frac{1}{2}\alpha C_2 T_2 \mu + \alpha\left(\frac{1}{2}C_1 + C_2\right)T_1\mu + \alpha C_2 k\sigma\sqrt{\tau_2} + \alpha(C_1 + C_2)k\sigma\sqrt{\tau_1}$$

$$s.t.\quad \sum_{j=1}^{O_1} t_{1j}y_{1j} - T_1 = 0\ ;\quad \sum_{j=1}^{O_1} c_{1j}y_{1j} - C_1 = 0\ ;\ xC_1 - C_2 = 0\ ;$$

$$xT_1 - T_2 = 0\quad \tau_2 = T_2 - S_2\ ;\quad \tau_1 = S_2 + T_1\ ;$$

$$\tau_i \geq 0\quad \text{for } i = 1, 2;\quad S_2 \geq 0$$

$$\sum_{j=1}^{O_1} y_{1j} = 1 \text{ where } y_{1j} \in \{0, 1\} \text{ for } 1 \leq j \leq O_1$$

The substantive difference between **P** and **P'** involves removing the indicator variables for stage 2 and adding the constraints $xC_1 - C_2 = 0$and $xT_1 - T_2 = 0$. This change forces the chosen option at stage 2 to always be a constant x of stage 1's chosen option. While our approach for simultaneously changing the options at both stages entails simplifying the option selection problem, there is precedent for this simplification in practice. First, for two-echelon supply chains consisting of production as the upstream echelon and distribution as the downstream echelon, it is not uncommon for the production activity to be measured in months while the distribution activity is measured in days. Second, Mandal (2004) and Moody (2006) both provide multiple examples of supply chain restructuring where options at different stages are changed in the same direction. Finally, it is consistent with the notion of synchronizing the supply chain, which was analytically demonstrated to hold for selecting candidates at a single stage.

In **P'**, when $x \geq 1$, it is optimal for stage 2 not to hold safety stock, and the supply chain configuration cost in the incumbent network can be expressed as

$$f(c_{11}, t_{11}) = \alpha(1 + x)c_{11}k\sigma\sqrt{(1 + x)t_{11}} + \frac{1}{2}\alpha x^2 c_{11}t_{11}\mu + \frac{1}{2}\alpha(2x + 1)c_{11}t_{11}\mu + (1 + x)\beta c_{11}\mu.$$

$f(c_{11}, t_{11})$ is a concave function, allowing us to define the following ratio to compare options,

$$R = \alpha(1 + x)c_{11}k\sigma\frac{1}{2\sqrt{(1 + x)t_{11}}} + \frac{1}{2}\alpha c_{11}\mu(x + 1)\Big/\alpha k\sigma\sqrt{(1 + x)t_{11}} + \frac{1}{2}\alpha(x + 1)t_{11}\mu + \beta\mu,$$

Theorem 6 formally provides the sufficient condition for accepting candidate options at both stages.

Theorem 6. *When considering candidate options* (c_{1j},t_{1j}) *at stage 1 and* (xc_{1j},xt_{1j}) *at stage 2 with* $x \geq 1$, *we accept Type II candidate options if* $\Delta C/\Delta T < R$ *and we accept Type IV candidates if* $\Delta C/\Delta T > R$.

We can now derive insights from the functional form of R.

Corollary 3. *R is an increasing function of c_{11}, α; and a decreasing function of t_{11} and β.*

The explanation for Corollary 3 is consistent with that of Corollaries 1 and 2.

We next compare the threshold for replacing the incumbent option at both stages with only changing the incumbent at one stage. We define $\tilde{R}_{11}$ and $\tilde{R}_{21}$ as the calculation of R_{11} and R_{21} with the assumption that the incumbent option's cost and leadtime at stage 2 is a constant x times the cost and leadtime at stage 1, thereby producing.

$$\tilde{R}_{11} = \alpha(1 + x)c_{11}k\sigma\frac{1}{2\sqrt{(1 + x)t_{11}}} + \frac{1}{2}\alpha(2x + 1)c_{11}\mu\Big/\alpha k\sigma\sqrt{(1 + x)t_{11}} + \frac{1}{2}\alpha t_{11}\mu + \beta\mu \text{ and.}$$

$$\tilde{R}_{21} = \alpha(1 + x)c_{11}k\sigma\frac{1}{2\sqrt{(1 + x)t_{11}}} + \frac{1}{2}\alpha x c_{11}\mu\Big/\alpha k\sigma\sqrt{(1 + x)t_{11}} + \frac{1}{2}(1 + 2x)\alpha t_{11}\mu + \beta\mu.$$

Comparing these thresholds to R, we have the following theorem.

Theorem 7. $\tilde{R}_{21} < R < \tilde{R}_{11}$

Theorem 7 shows that accepting Type II candidates is least financially viable when we only change the incumbent option at stage 2, it is more viable if we change both stages and it is the most viable if we only change the incumbent option at stage 1. Theorem 5 explains why $\tilde{R}_{21} < \tilde{R}_{11}$. The reason R lies between $\tilde{R}_{21}$ and $\tilde{R}_{11}$ is that choosing Type II candidates at both stages definitely increases COGS in the supply chain and the increased cost at stage 2's option adds to stage 1's cumulative cost. Therefore, comparing with the scenario of introducing the Type II candidate to only stage 1, it is harder for a decrease in the other cost components to offset this cost increase. However, when compared to the scenario of introducing the candidate to only stage 2, the advantage of introducing the Type II candidates to both stages lies in the fact that the total pipeline stock cost at stage 1 may decrease due to the leadtime reduction, while changing to a Type II candidate only at stage 2 guarantees the pipeline stock cost at stage 1 will increase. Also, when both stages select Type II candidate options, the supply chain may carry much less safety stock inventory than if only stage 2 adopts a Type II candidate option.

Theorem 7 also helps justify why managers may only change one portion of their supply chain (as in Sections 4.1 and 4.2), documenting that in the supply chain configuration problem changing the option at one stage, particularly a downstream stage, can reduce cost more than changing the options at both stages. Theorem 7 codified two sourcing results that the strategic sourcing teams knew but still viewed as counterintuitive before seeing this analysis. First, they knew in practice that it was extremely difficult to simply replace a part with one that was higher cost even with a significantly shorter leadtime than its incumbent. Second, they would often switch to air transport for finished goods locations but rarely upstream in the supply chain. This analysis put the downstream effects of these alternatives in a clearer light.

5. Assuming a functional form between an option's cost and leadtime

In all the previous sections, we make no assumption concerning the relationship between an option's cost and leadtime. However, assuming a functional relationship between an option's cost and leadtime is worth briefly considering because it allows us to abstract the problem enough to yield more general insights concerning the relative value of high cost short leadtime versus low cost long leadtime options. As a general statement, the strategic sourcing teams were most interested in understanding whether Type II or Type IV options were the most likely to be accepted. They wanted to reduce their search space to gain efficiency and focus more resources on the candidate options that could be viable. By allowing a functional form between cost and leadtime, options exist as a continuous function so the optimal cost-leadtime relationship can be found. While strategic sourcing understands that options are discrete, this analysis was viewed as an important thought exercise to understand the underpinnings of the problem.

Project management literature has extensive discussion on cost-time relationships. Robinson (1975) optimizes project scheduling with the objective function incorporating both the project leadtime and cost, and they assume that the cost is deterministic and nonincreasing over the domain of leadtime. Teece (1977) empirically estimates the relationship between cost and time as $C = Ve^{\varphi/((t/a)-1)}$ where C is cost, t leadtime, and V, α, and ϕ are project-specific parameters. Hill and Khosla (1992) develop a conceptual framework to consider the tradeoff between cost and time; the functional relationship is defined as $V(L) = \alpha L^{-\beta}$, where L is leadtime, and $V(L)$ is the unit cost. The parameter β captures lead-time elasticity. When other parameters are fixed, increasing β reduces the cost of shortening leadtime.

These results codify intuition that reducing time increases cost in a certain functional way. Therefore, in this subsection, we follow Hill and Khosla (1992), and use $c_{11} = Mt_{11}^{-n}$. Maintaining our assumptions from Section 4.3, we conduct our analysis on four ranges of n: $0 < n < ½$, $n = ½$, $½ < n < 1$, and $n \geq 1$.

When $0 < n < ½$, we can prove that the supply chain configuration cost is a convex function of c_{11}, and there exists a minimum, although the analytical expression of this minimum can not be obtained. As for the case of $½ < n < 1$, neither the convexity nor the concavity of the supply chain configuration cost can be proved, and no general results can be derived. Theorem 8 considers the range $n \geq 1$.

Theorem 8. *When $c_{11} = Mt_{11}^{-n}$ with $n \geq 1$, supply chain configuration cost is minimized by choosing the cheapest available option.*

Theorem 8 is a justification for many company's lowest-cost-searching strategy in configuring their supply chain, such as HP's ABC (Absolute Best Cost) strategy described in Moody (2006). Tying this back to our understanding of Type II and IV options, if the cheapest available option is not the incumbent option, then it will never be optimal to choose a Type II option and any available Type IV option will lower supply chain cost, with the lowest-cost Type IV option available reducing supply chain configuration cost the most. Theorem 9 addresses the case where $n = ½$.

Theorem 9. *When $c_{11} = M/\sqrt{t_{11}}$, total supply chain configuration cost is a convex function of c_{11} and the global minimum is reached when $C^* = \sqrt{\alpha M^{1/n}(1 + x)/2\beta}$*

Fig. 4 shows the supply chain configuration cost as a function of the cost at stage 1 when $n = ½$. It is noteworthy that when neither x nor M is too large, C^* tends to be very small because α is almost surely less than one and β is usually significantly greater than α. Therefore, it is very likely C^* could never be reached in practice, and the first option existing would be greater than C^*. This would be the cheapest option available. And if the incumbent option had a cost greater than this value, then the optimal choice would be a Type IV candidate.

Given the caveats associated with how reasonable it is to assume that all options fall along a functional form defining the relationship between an option's cost and time, it is interesting in two cases ($n \geq 1$ and $n = ½$) it is optimal to choose the longest leadtime and cheapest option available. This demonstrates the power and problem of employing a functional form between cost and time. The power is that we can analytically calculate thresholds and prove behaviors. The problem is that we push to the extremes allowed by the functions. In practice, the discrete nature of options prevents a blanket rule of always choosing the cheapest option, but the results confirm the intuition that choosing the cheapest option is a reasonable heuristic.

6. Conclusions and applications

In this paper, we consider the supply chain configuration problem for a two-stage serial line.

We employ a quadrant-based view of the cost and leadtime differences between incumbent and candidate options, and derive sufficient conditions to dictate when candidate options lower supply chain configuration cost relative to incumbent options. We analytically prove that if COGS is more significant than safety and pipeline stock cost, the optimal set of Type II candidate options will decrease relative to Type IV candidate options. We prove there is benefit to synchronizing the supply chain, where both stages adopt low cost long leadtime options or high cost short leadtime options. We also prove the impact that system parameters and the position of the stage in the supply chain have on the optimal supply chain configuration.

We prove that the benefit of employing Type II candidate options at both stages is less beneficial than employing Type II candidates at only stage 1 but more beneficial than only employing Type II candidates at stage 2. Lastly, we assume a functional relationship between an option's cost and time, demonstrating that two functional forms relating time and cost dictate the preference for Type IV options.

Here is an example of how these insights can be applied in practice. As the Gillette Company (now part of Procter & Gamble) prepared to launch Mach3 Power razor in 2004, they built their supply chain based on the existing Mach3 razor supply chain, which had a manufacturing stage offshore and a transportation

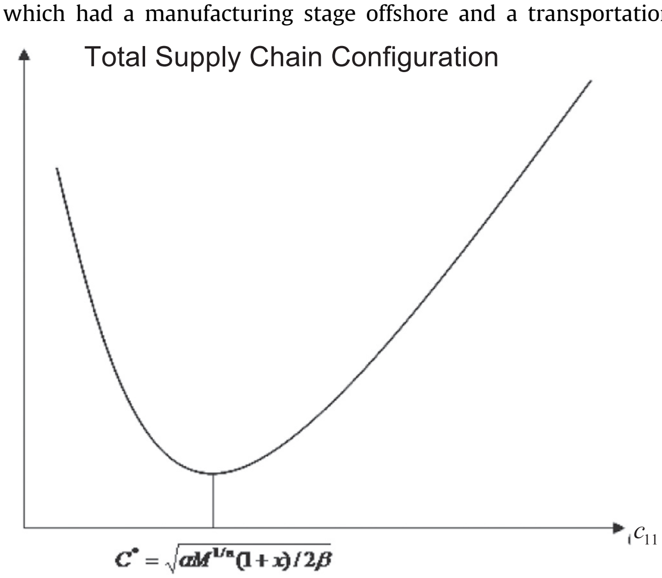

Fig. 4. Supply chain configuration cost as a function of c_{11}, when n = ½.

stage employing sea shipping to supply distribution centers in North America. One key difference between the Mach3 Power razor and Mach3 razor is that the Mach3 Power used a power handle which was significantly more expensive to produce than the Mach3's manual handle. After assessing the total configuration cost of their supply chain, P&G decided to use air shipping for Mach3 Power handles, instead of the traditional sea shipping method for Mach3 handles. In this situation, the presence of a higher cost option at the upstream stage made it optimal to choose a higher cost shorter leadtime candidate option at the downstream stage. This is consistent with the synchronization concepts proposed in our work, as described in Sections 4.1 and 4.2.

This paper is an early attempt at rigorously characterizing the supply chain configuration problem. As such, it has significant room for future development. The two-stage serial line supply chain is a good starting point but the work will benefit from extending results to more generalized network structures. It could be interesting to enrich the definition of an option beyond a cost-time pairing to incorporate other dimensions like quality. This would allow a richer definition of supply chain configuration cost. Similarly, extending the work to a stochastic service setting where the service time was not guaranteed would be interesting; such a significant change would likely require augmenting the state space.

Finally, we assumed any fixed costs associated with selecting a new option were not significant. There are surely cases where fixed costs are significant. If the fixed cost is simply an option-specific constant that will change the specific thresholds but will not change the general insights.

Appendix. Proofs

Proof of Lemma 1

1) To prove $h_1(c_{21}, t_{21}) = \alpha(c_{11} + c_{21})k\sigma\sqrt{t_{11} + t_{21}} + \frac{1}{2}\alpha c_{21}t_{21}\mu + \frac{1}{2}\alpha(2c_{21} + c_{11})t_{11}\mu + \beta(c_{11} + c_{21})\mu$ is quasi-concave, we first show that the contour of h_1, i.e. $h_1(c_{21}, t_{21}) = A$, expressed as $c_{21} = Q(t_{21})$, is convex function of t_{21}. By definition, we have $c_{21} = Q(t_{21}) = \frac{A - \alpha k\sigma c_{11}\sqrt{t_{11}+t_{21}} - \frac{1}{2}\alpha c_{11}t_{11}\mu - \beta c_{11}\mu}{\alpha k\sigma\sqrt{t_{11}+t_{21}} + \frac{1}{2}\alpha t_{21}\mu + \alpha t_{11}\mu + \beta\mu} > 0$. Taking the second derivative of $Q(t_{21})$, we have

$$\frac{\partial^2 Q(t_{21})}{\partial t_{21}^2} = \frac{\alpha k\sigma c_{11}}{4(t_{11}+t_{21})^{3/2}\left(\alpha k\sigma\sqrt{t_{11}+t_{21}} + \frac{1}{2}\alpha t_{21}\mu + \alpha t_{11}\mu + \beta\mu\right)} + \frac{\alpha k\sigma c_{11}\left(\frac{1}{2}\frac{\alpha k\sigma}{\sqrt{t_{11}+t_{21}}} + \frac{1}{2}\alpha\mu\right)}{\sqrt{t_{11}+t_{21}}\left(\alpha k\sigma\sqrt{t_{11}+t_{21}} + \frac{1}{2}\alpha t_{21}\mu + \alpha t_{11}\mu + \beta\mu\right)^2} + \frac{2\left(A - \alpha k\sigma c_{11}\sqrt{t_{11}+t_{21}} - \frac{1}{2}\alpha c_{11}t_{11}\mu - \beta c_{11}\mu\right)\left(\frac{1}{2}\frac{\alpha k\sigma}{\sqrt{t_{11}+t_{21}}} + \frac{1}{2}\alpha\mu\right)^2}{\left(\alpha k\sigma\sqrt{t_{11}+t_{21}} + \frac{1}{2}\alpha t_{21}\mu + \alpha t_{11}\mu + \beta\mu\right)^3} + \frac{\frac{1}{4}\left(A - \alpha k\sigma c_{11}\sqrt{t_{11}+t_{21}} - \frac{1}{2}\alpha c_{11}t_{11}\mu - \beta c_{11}\mu\right)\alpha k\sigma}{\left(\alpha k\sigma\sqrt{t_{11}+t_{21}} + \frac{1}{2}\alpha t_{21}\mu + \alpha t_{11}\mu + \beta\mu\right)^2(t_{11}+t_{21})^{3/2}} > 0$$

So $Q(t_{21})$ is a convex function of t_{21}. As a result, the epigraph of $Q(t_{21})$ is a convex set. Since $h_1(c_{21}, t_{21})$ is an increasing function of c_{21} and t_{21}, so the epigraph of $Q(t_{21})$ is equivalent to $S = \{(c_{21}, t_{21}), s.t.\ h_1(c_{21}, t_{21}) \geq A\}$, which is convex. By definition, $h_1(c_{21}, t_{21})$ is quasi-concave.

2) Using the same technique as we used for $h_1(c_{21}, t_{21})$, we can also show that $h_2(c_{21}, t_{21})$ is quasi-concave.

3) Therefore, $h(c_{21}, t_{21}) = \min(h_1(c_{21}, t_{21}), h_2(c_{21}, t_{21}))$ is quasi-concave.

Proof of Theorem 1 and Theorem 2

We show the proof for the scenario where only stage 1 holds safety stock inventory in the incumbent network, the proof for both stages holding safety stock is the same with $h_1(c_{21}, t_{21})$ replaced by $h_2(c_{21}, t_{21})$.

1) We assume that the incumbent option at stage 2 has cost c_{21} and leadtime t_{21} and the Type II candidate option has cost c_{2j} and leadtime t_{2j}. If we accept the candidate option, then $h(c_{21}, t_{21}) = h_1(c_{21}, t_{21})$ is changed along the direction $d = (c_{2j} - c_{21}, t_{2j} - t_{21})' = (\Delta C, -\Delta T)'$ Then according to the property of quasi-concave function, $h(c_{2j}, t_{2j}) < h(c_{21}, t_{21})$ if $\nabla h(c_{21}, t_{21})'d = \nabla h_1(c_{21}, t_{21})'d < 0$, i.e.

$$\left[\alpha k\sigma\sqrt{t_{11}+t_{21}} + \frac{1}{2}\alpha t_{21}\mu + \alpha t_{11}\mu + \beta\mu\right]\Delta C + \left[\alpha(c_{11}+c_{21})k\sigma\frac{1}{2\sqrt{t_{11}+t_{21}}} + \frac{1}{2}\alpha c_{21}\mu\right]\left[-\Delta T\right] < 0 \quad \text{,i.e.}$$

$$\frac{\Delta C}{\Delta T} < \left\{\alpha(c_{11}+c_{21})k\sigma\frac{1}{2\sqrt{t_{11}+t_{21}}} + \frac{1}{2}\alpha c_{21}\mu\right\} \Big/ \left\{\alpha k\sigma\sqrt{t_{11}+t_{21}} + \frac{1}{2}\alpha t_{21}\mu + \alpha t_{11}\mu + \beta\mu\right\}.$$

Therefore, we accept this Type II candidate option if $\Delta C/\Delta T < R_{21}$.

2) When the candidate option with cost c_{2j} and leadtime t_{2j} is of Type IV, then $h(c_{21}, t_{21}) = h_1(c_{21}, t_{21})$ is changed along the direction $d = (c_{2j} - c_{21}, t_{2j} - t_{21})' = (-\Delta C, \Delta T')$. According to quasi-concave function property, $h(c_{2j}, t_{2j}) < h(c_{21}, t_{21})$ if $\nabla h(c_{21}, t_{21})'d = \nabla h_1(c_{21}, t_{21})'d < 0$, , i.e.

$$\frac{\Delta C}{\Delta T} > \left\{\alpha(c_{11}+c_{21})k\sigma\frac{1}{2\sqrt{t_{11}+t_{21}}} + \frac{1}{2}\alpha c_{21}\mu\right\} \Big/ \left\{\alpha k\sigma\sqrt{t_{11}+t_{21}} + \frac{1}{2}\alpha t_{21}\mu + \alpha t_{11}\mu + \beta\mu\right\}.$$

So we accept the Type IV candidate option if $\Delta C/\Delta T > R_{21}$.

Proof of Corollary 1

We only show the proof for R_{21}. The proof for R_{22} is similar to R_{21}.

For R_{21}, we only need to prove the t_{11} and t_{21} results, and other conclusions are obvious.

$$\frac{\partial R_{21}}{\partial t_{11}} = -\frac{\alpha(c_{11}+c_{21})k\sigma}{4(t_{11}+t_{21})^{3/2}(\alpha k\sigma\sqrt{t_{11}+t_{21}} + \frac{1}{2}\alpha t_{21}\mu + \alpha t_{11}\mu + \beta\mu)} - \frac{(\alpha(c_{11}+c_{21})k\sigma/\sqrt{t_{11}+t_{21}} + \alpha c_{21}\mu)(\alpha k\sigma/\sqrt{t_{11}+t_{21}} + 2\alpha\mu)}{4(\alpha k\sigma\sqrt{t_{11}+t_{21}} + \frac{1}{2}\alpha t_{21}\mu + \alpha t_{11}\mu + \beta\mu)^2} < 0$$

Therefore, R_{21} is a decreasing function of t_{11}. Similarly, as for t_{21}

$$\frac{\partial R_{21}}{\partial t_{21}} = -\frac{\alpha(c_{11}+c_{21})k\sigma}{4(t_{11}+t_{21})^{3/2}(\alpha k\sigma\sqrt{t_{11}+t_{21}} + \frac{1}{2}\alpha t_{21}\mu + \alpha t_{11}\mu + \beta\mu)} - \frac{(\alpha(c_{11}+c_{21})k\sigma/\sqrt{t_{11}+t_{21}} + \alpha c_{21}\mu)(\alpha k\sigma/\sqrt{t_{11}+t_{21}} + \alpha\mu)}{4(\alpha k\sigma\sqrt{t_{11}+t_{21}} + \frac{1}{2}\alpha t_{21}\mu + \alpha t_{11}\mu + \beta\mu)^2} < 0$$

So R_{21} is also a decreasing function of t_{21}.

Proof of Theorem 3 and Theorem 4

The steps are similar to Theorem 1 and Theorem 2.

Proof of Corollary 2

The proof steps are similar to Corollary 1.

Proof of Theorem 5

$$R_{21} = \frac{\alpha(c_{11}+c_{21})k\sigma\frac{1}{2\sqrt{t_{11}+t_{21}}}+\frac{1}{2}\alpha c_{21}\mu}{\alpha k\sigma\sqrt{t_{11}+t_{21}}+\frac{1}{2}\alpha t_{21}\mu+\alpha t_{11}\mu+\beta\mu} \text{ and } R_{11} = \frac{\alpha(c_{11}+c_{21})k\sigma\frac{1}{2\sqrt{t_{11}+t_{21}}}+\frac{1}{2}\alpha(2c_{21}+c_{11})\mu}{\alpha k\sigma\sqrt{t_{11}+t_{21}}+\frac{1}{2}\alpha t_{11}\mu+\beta\mu}.$$

We have $\alpha k\sigma\sqrt{t_{11}+t_{21}} + \frac{1}{2}\alpha t_{21}\mu + \alpha t_{11}\mu + \beta\mu > \alpha k\sigma\sqrt{t_{11}+t_{21}} + \frac{1}{2}\alpha t_{11}\mu + \beta\mu$ and $\alpha(c_{11}+c_{21})k\sigma\frac{1}{2\sqrt{t_{11}+t_{21}}} + \frac{1}{2}\alpha c_{21}\mu < \alpha(c_{11}+c_{21})k\sigma\frac{1}{2\sqrt{t_{11}+t_{21}}} + \frac{1}{2}\alpha(2c_{21}+c_{11})\mu$, so $R_{21} \le R_{11}$.

The way to compare R_{22} and R_{12} is similar. □

Proof of Theorem 6

The steps are similar to Theorem 1 and Theorem 2.

Proof of Corollary 3

The steps are similar to Corollary 1.

Proof of Theorem 7

In the case of $x \ge 1$, when we compare denominators we have $\tilde{R}_{21} < R < \tilde{R}_{11}$, and when we compare the numerators we have $\tilde{R}_{21} > R > \tilde{R}_{11}$. Therefore, $\tilde{R}_{21} < R < \tilde{R}_{11}$.

Proof of Theorem 8

Assuming the cost at stage 1 is a function of its leadtime, and they satisfy the functional relationship of $c_{11} = Mt_{11}^{-n}$, and $n \ge 1$. Then the total supply chain configuration cost function can be expressed as

$$f(c_{11}, (c_{11}/M)^n) = \alpha(1+x)k\sigma\sqrt{(1+x)M^{1/n}c_{11}^{(2n-1)/n}} + \frac{1}{2}(x^2+2x+1)\alpha M^{1/n}c_{11}^{(n-1)/n}\mu + (1+x)\beta c_{11}\mu.$$

Denoting this function as $g(c_{11})$, then we have

$$\frac{\partial g(c_{11})}{\partial c_{11}} = \left(1-\frac{1}{2n}\right)\alpha(1+x)k\sigma c_{11}^{-1/2n}\sqrt{(1+x)M^{1/n}} + \frac{1}{2}\left(\frac{n-1}{n}\right)(x^2+2x+1)\alpha M^{1/n}c_{11}^{-1/n}\mu + (1+x)\beta\mu$$
$$> 0 \frac{\partial^2 g(c_{11})}{\partial c_{11}^2} = \left(\frac{1}{4n^2}-\frac{1}{n}\right)\alpha(1+x)k\sigma c_{11}^{-1-1/2n}\sqrt{(1+x)M^{1/n}} + \frac{1}{2}\left(\frac{1}{n^2}-\frac{1}{n}\right)(x^2+2x+1)\alpha M^{1/n}c_{11}^{-1-1/n}\mu < 0$$

Therefore, $g(c_{11})$ is an increasing concave function. □

Proof of Theorem 9

Assuming the cost and leadtime at stage 1 satisfy $c_{11} = Mt_{11}^{-1/2}$, and $n \ge 1$, total supply chain configuration cost function can be expressed as $f(c_{11}, t_{11}) = f(c_{11}, (c_{11}/M)^2) = \alpha(1+x)k\sigma\sqrt{(1+x)M^2} + \frac{1}{2}(x^2+2x+1)\alpha M^2 c_{11}^{-1}\mu + (1+x)\beta c_{11}\mu$

Again, denoting this function as $g(c_{11})$, taking the second derivative of g, we have $\frac{\partial^2 g(c_{11})}{\partial c_{11}^2} = (x^2+2x+1)\alpha M^{1/n}c_{11}^{-3}\mu > 0$. Thus $g(c_{11})$ is a convex function of c_{11} and the minimum is reached when $\frac{\partial g(c_{11})}{\partial c_{11}} = -\frac{1}{2}(x^2+2x+1)\alpha M^{1/n}c_{11}^{-2}\mu + (1+x)\beta\mu = 0$, i.e. $C^* = \sqrt{\frac{(x+1)M^{1/n}\alpha}{2\beta}}$.

References

Anupindi, R., Akella, R., 1993. Diversification under supply uncertainty. Manag. Sci. 39, 944–963.

Babich, V., Burnetas, A.N., Ritchken, P.H., 2007. Competition and diversification effects in supply chains with supplier default risk. Manuf. Serv. Oper. Manag. 9 (2), 123–146.

Bertazzi, L., Speranza, M.G., Ukovich, W., 2000. Exact and heuristic solutions for a shipment problem with given frequencies. Manag. Sci. 46, 973–988.

Bossert, J.M., Willems, S.P., 2007. A periodic review modeling approach for guaranteed service supply chains. Interfaces 37 (5), 420–435.

Boute, R.N., Van Mieghem, J.A., 2011. Global Dual Sourcing and Order Smoothing, Working Paper, (32 pp.).

Callioni, G., de Montgros, X., Slagmulder, R., Van Wassenhove, L.N., Wright, L., 2005. Inventory-driven costs. Harv. Bus. Rev. 99 (2) 99–99.

Cetinkaya, S., Lee, C., 2000. Stock replenishment and shipment scheduling for vendor-managed inventory systems. Manag. Sci. 46, 217–232.

Graves, S.C., Willems, S.P., 2003. Supply chain design: safety stock placement and supply chain configuration. Handbooks in Operations Research and Management Science vol 11. North-Holland Publishing Company, Amsterdam.

Graves, S.C., Willems, S.P., 2005. Optimizing the supply-chain configuration for new products. Manag. Sci. 51, 1165–1180.

Graves, S.C., Willems, S.P., 2008. Strategic inventory placement in supply chains: nonstationary demand. Manuf. Serv. Oper. Manag. 10 (2), 278–287.

Gross, D., Soriano, A., 1969. The effect of reducing leadtime on inventory levels – simulation analysis. Manag. Sci. 16, B-61–B76.

Hill, A.V., Khosla, I.S., 1992. Models for optimal lead time reduction. Prod. Oper. Manag. 1, 185–197.

Huggins, E.L., Olsen, T.L., 2003. Supply chain management with guaranteed delivery. Manag. Sci. 49, 1154–1167.

Jain, A., Groenevelt, H., Rudi, N., 2010. Continuous review inventory model with dynamic choice of two freight modes with fixed costs. Manuf. Serv. Oper. Manag. 12 (1), 120–139.

Lawson, D., Porteus, E.L., 2000. Multistage inventory management with expediting. Oper. Res. 48, 878–893.

Mandal, K, 2004. How Eveready Energized its Supply Chain. Businessline. Chennai: Jan 19, P1.

Minner, S., 2003. Multiple-supplier inventory models in supply chain management: a review. Int. J. Prod. Econ. 81–82, 265–279.

Moody, P.E., 2006. With supply management, technology rules!. Supply Chain Manag. Rev. 10 (4), 41–49.

Muharremoglu, A., Tsitsiklis, J.N., 2003. Dynamic Leadtime Management in Supply Chains, unpublished manuscript.

Robinson, D.R., 1975. A dynamic programming solution to cost-time tradeoff for CPM. Manag. Sci. 22, 158–166.

Simpson, K.F., 1958. In-process inventories. Oper. Res. 6, 863–873.

Teece, D., 1977. Time-cost tradeoffs: elasticity estimates and determinants for international technology transfer projects. Manag. Sci. 23, 830–837.

Thomas, D.J., Tyworth, J.E., 2006. Pooling lead-time risk by order splitting: a critical review. Transp. Res. Part E: Logist. Transp. Rev. 42 (4), 245–257.

Tomlin, B., 2009. Impact of supply learning when suppliers are unreliable. Manuf. Serv. Oper. Manag. 11 (2), 192–209.

Willems, S.P., 2008. Data set: real-world multi-echelon supply chains used for inventory optimization. Manuf. Serv. Oper. Manag. 10, 19–23.