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Achievement in Operations Research

Inventory Optimization at Procter & Gamble: Achieving Real Benefits Through User Adoption of Inventory Tools

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Over the past 10 years, Procter & Gamble has leveraged its cross-functional organizational structure with operations research to reduce its inventory investment. Savings were achieved in a two-step process. First, spreadsheet-based inventory models locally optimized each stage in the supply chain. Because these were the first inventory tools installed, they achieved significant savings and established P&G's scientific inventory practices. Second, P&G's more complex supply chains implemented multiechelon inventory optimization software to minimize inventory costs across the end-to-end supply chain. In 2009, a tightly coordinated planner-led effort, supported by these tools, drove \$1.5 billion in cash savings. Although case studies show the mathematics employed, of equal importance is the presentation of the planning process that facilitates inventory management and the decision tree that matches a business to the optimal inventory tool depending on the requirements of the business. Today, more than 90 percent of P&G's business units (about \$70 billion in revenues) use either single-stage (70 percent) or multiechelon (30 percent) inventory management tools. Plans are underway to increase the use of multiechelon tools to manage 65 percent of P&G's supply chains in the next three years.

Key words: industries: consumer packaged goods; inventory/production: applications, multi-item/echelon/stage; dynamic programming: applications.

Procter & Gamble (P&G), founded in 1837, is a leading global consumer products company with \$76.7 billion in sales in 180 countries. Four billion times a day, P&G touches the lives of people around the world through its familiar brand names—Tide, Crest, Gillette, Pantene, and more than 200 others. It competes in 26 distinct product-category markets such as hair care, paper towels, cosmetics, skin care, oral care, blades and razors, diapers, and fabric care.

Inventory management in a company of P&G's size, scale, and complexity requires leveraging the right people, organization, and tools. By marrying the appropriate operations research (OR) techniques with a unique planning-organization structure, P&G has achieved two step-change improvements in inventory levels. The first improvement came from the broad application of spreadsheet-based inventory models; this work produced four tools that locally optimize different portions of the supply chain. The second improvement integrated multiechelon inventory software in P&G's more-complex supply chains. In 2009, these tools, implemented through a well-coordinated planning community, were instrumental in driving \$1.5 billion in cash savings, while maintaining or increasing service levels.

This paper details how P&G implemented advanced inventory optimization technology in more than 90 percent of its businesses to reduce its inventory investment significantly over the past 10 years. Although case studies show the mathematics employed, of equal importance is the presentation of the planning process that facilitates inventory management and the decision tree that matches a business to the optimal inventory tool depending on business requirements.

How P&G Manages Inventory

The goal of P&G's logistics planning effort is to delight our consumers and shoppers with world-class products that are designed and packaged to suit their needs, available wherever they shop, and priced to represent the best value for them.

P&G's logistics planning workforce consists of over 5,000 individuals who plan material supply, capacity, inventory, and logistics for the company's 500 supply chains. The total supply chain network

comprises 145 P&G-owned manufacturing facilities and 300 contract manufacturers, resulting in over 6,900 unique product-category market combinations served. Each supply chain requires well-coordinated efforts based on best-available information, communication, and planning tools. Production batch sizes, planning reaction times, order policies, replenishment timing, physical facility restrictions, customer order patterns, modes of transportation, new-product introductions, promotions on existing brands, assortment management, and regulatory requirements combine to present complex challenges and trade-offs.

Each of P&G's three global business units (GBUs), Beauty & Grooming (\$26.3 billion), Household Care (\$37.3 billion), and Health & Well-Being (\$14.4 billion), would be large enough to be placed on the 2009 Fortune 200 list. As such, each has an internal support and line organization responsible for planning that takes into account the unique constraints of the business.

To drive scale and efficiency through P&G's supply network operations, horizontal process networks (HPNs) (see Figure 1) span across all of P&G. HPNs define, manage, and execute standard work processes across P&G's business operating units.

For example, the category supply planning HPN is responsible for developing plans that optimize the supply network's capability to deliver products as required by the customer at lowest delivered cost. This involves balancing numerous constraints against objectives that can conflict (e.g., defining capacity requirements to ensure that targeted utilization rates are met, while ensuring that sufficient inventories are maintained in the right quantities and at the right place). Planners also must ensure that manufacturing resources are planned in a manner that supports strategic objectives and delivers against customer requirements.

HPN members include GBU representatives, information technology (IT) service managers, and global business and technical experts such as the supply chain OR group. The teams are responsible for determining the scope of a globally standard solution set consisting of work processes and tools. The teams also govern the use of the solutions—maintaining standards and determining innovation requirements for

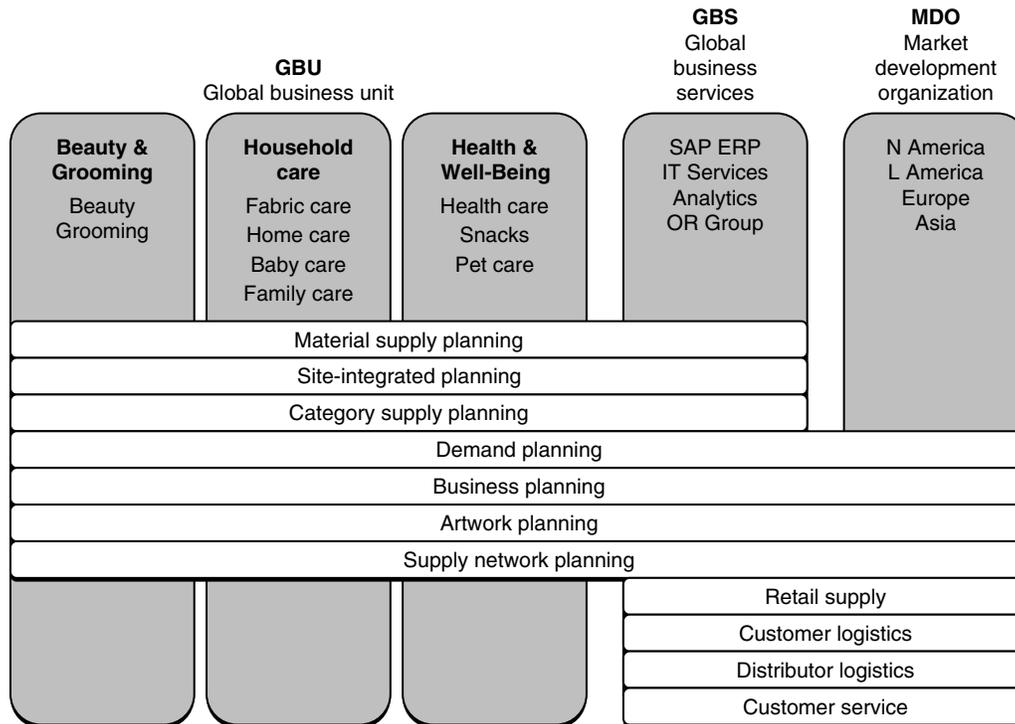


Figure 1: P&G’s 11 HPNs enable cross-business unit best practices and scalability for P&G.

improving the current solution and supporting the company’s growth plans.

These global networks provide the backbone to drive scalability and efficiency for the company and are a primary reason why P&G is ranked as one of the best-managed supply chains in the world. It has won numerous best-practice awards, including multiple consecutive top-five placements in AMR Research’s list. Through the HPNs, planners are continually trained to use planning tools and systems to help meet business objectives regarding cost, cash, and service management.

For a company with P&G’s product depth and breadth, a one-size-fits-all inventory strategy is not optimal. Capacity utilization of high-speed papermaking operations, in which capital and logistics costs demand high utilization and turnover for profitability, differs dramatically from hair-care batch-making operations in which many stock-keeping units (SKUs) are made and distributed through a multiple-tier distribution network. Different businesses require different modeling methods and tools to determine the

correct inventory levels. In recognition of this need for different inventory models, P&G has deployed a framework (see Figure 2) that assigns the appropriate inventory technology to any business based on its operating characteristics and complexity. In the next paragraph, we describe the framework and its applicable terms.

The primary selection criterion is network complexity—whether or not the planner is setting inventory targets at several echelons in the supply chain. As we will show, the majority of supply chains in the Beauty & Grooming GBU are in this category. These supply chains span multiple echelons and benefit tremendously from multiechelon inventory optimization. Depending on the workflow defined by the planner, the technology employed is a third-party software solution from Logility, Inc.—either Voyager Inventory Strategy Optimization (VISO) or Voyager Inventory Target Optimization (VITO). Alternatively, many of the supply chains in the Household Care GBU are planned at a single echelon. A single-stage inventory tool has been customized

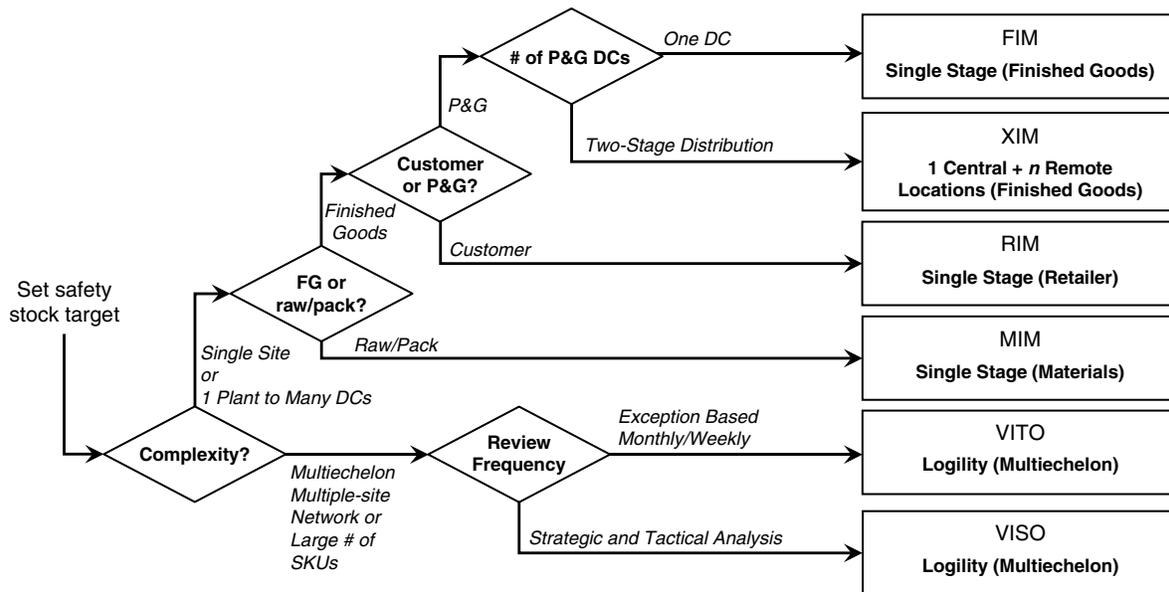


Figure 2: P&G’s OR group has developed a decision tree for its businesses to follow to match the correct inventory tool to the appropriate business conditions. Complex networks, defined by multiple echelons or many SKUs, employ multiechelon inventory optimization models, whereas single-stage settings employ customized single-stage inventory management solutions tailored to the operating environment.

to each echelon’s most important focus—materials (MIM), finished goods held at a P&G plant location (FIM), or at a distribution point with several remote stocking locations (XIM). Custom variations of the model have been developed for retail customer needs (RIM) and to better predict prebuild inventory levels for new products. To demonstrate the breadth of these tools, we focus on two implementations that correspond to the nodes in Figure 2: single-stage inventory models in Western Europe and multiechelon models in North America.

Single-Stage Inventory Models

P&G has a long history of applying inventory management techniques. Its use of scientific inventory planning dates to the 1970s (Murphy 1975); however, the adoption of distribution requirements planning (DRP) systems in the late 1980s triggered P&G’s development of simple but robust models for setting inventory targets across a distribution network, one echelon at a time. P&G has successfully adopted single-stage inventory models in over 60 percent of its supply chains. Coverage by region varies from

about 50 percent in the Americas to over 80 percent in Europe, the Middle East, and Africa (EMEA) and Asia.

The single-stage models were developed by P&G’s internal OR group and implemented as end-user spreadsheets for use by planners. We refer to Farasyn et al. (2008) for a deeper discussion of our single-stage inventory models, including a review of the design choices, benefits, and weaknesses of spreadsheets as the technology platform for this type of application.

Business Benefits

Statistical inventory models avoid supply chain planner bias toward excessively high levels of safety stock. Indeed, safety stock at too low a level would quickly result in customer service incidents; the planner would fix this issue by choosing higher safety stock targets. However, when a planner manually sets a high safety stock target, no automatic correction mechanism is available to adjust the safety stock levels downward.

Across all regions and business units, we see a dramatic inventory reduction the first time the models are used for a specific supply chain (see Figure 3). For

Global business unit	Region	Model scope	Inventory model	Total inventory reduction (%)
Beauty & Grooming	North America	Skin-care plants	FIM	–50
Household Care	Western Europe	Amiens, France plant to Germany warehouse	XIM	–12
Household Care	Western Europe	Mechelen, Belgium plant to UK warehouse	XIM	–30
Multiple	Central and Eastern Europe	Regional plants and warehouses	FIM/XIM	–10
Household Care	Central and Eastern Europe	Moscow central warehouse	XIM	–10
Household Care	Asia	Pan-Asia Pringles	FIM	–12
Household Care	Asia	China Pringles	FIM	–10

Figure 3: Examples illustrate applying single-stage inventory models to various businesses at P&G.

example, the initial implementation of the extended single-stage model, XIM, on the Lenor fabric softener inventory produced at the plant in Amiens, France, yielded a realized inventory reduction of 12 percent. Figure 3 provides additional examples of the model's application. Although not exhaustive, it provides evidence of the model's robustness across a number of different business categories and regions.

In addition to the quantifiable results, there are intangible benefits. The single-stage models have created a common language for inventory discussions in the supply chain function. Planners have become more aware of the role and influence of the various inventory building blocks (e.g., safety, cycle, transit stocks). Inventory concepts are now an important part of the supply chain function's curriculum, helping planners to manage the complex trade-offs between capacity, inventory, and service.

Case Study: Fabric & Home Care, Western Europe

Despite growing business complexity, our single-stage models have enabled the Fabric & Home Care (F&HC) business in Western Europe to reduce its inventory and maintain high customer-service levels. This business unit comprises approximately 20 manufacturing and customization centers. Fabric care products include laundry granules, liquids and tablets, liquid tablets, bleach, and fabric enhancers. Home Care includes automatic and manual dish care, surface cleaners, and air care products.

Figure 4 tracks inventory and customer service measures over the 10-year period from fiscal year

(FY) 1999–2000 through FY2009–2010. Inventory is normalized on a 100-unit scale, and service is measured as case fill rate. Implementing the initial single-stage model (FIM) in the F&HC Western Europe businesses resulted in a 29 percent inventory decrease in the first five years. In subsequent years, in which the next-generation, eXtended Inventory Model (XIM) was introduced, an additional 10 percent inventory reduction was realized.

Following the initial dramatic improvement as a result of introducing the single-stage tool in FY1999–2000, service has remained stable and the F&HC business in Western Europe has achieved its 99.5 percent case fill-rate objective.

F&HC Western Europe realized this improvement in spite of growing complexity. Over the past

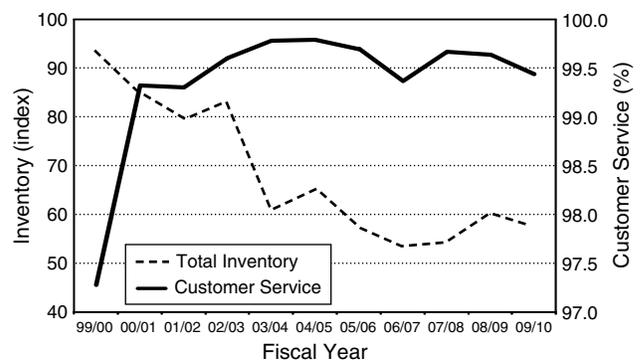


Figure 4: By implementing single-stage inventory models in F&HC Western Europe, total inventory decreased 36 percent; service rose above 98.5 percent.

10 years, it has had to deal with significant product proliferation—partly because of acquisitions and partly driven by new-product introductions and innovations. Although its number of finished goods has doubled (i.e., from under 2,500 to over 5,000) during these 10 years, inventory levels have held steady.

Multi-echelon Inventory Optimization in P&G's Beauty & Grooming GBU

Of P&G's three GBUs, Beauty & Grooming's products have the most complex supply chains. Several factors drive this complexity, all of which increase the benefits derived from multi-echelon inventory optimization. In comparison to the other GBUs, it has more finished goods SKUs, both for everyday items and promotional items. It also commonly customizes packaging to match a customer's unique requirements. In general, customized packaging has a lower unit volume per item, a higher sales price per item, and more echelons in the supply chain. Raw materials are often turned into intermediate products using a batch manufacturing process; these products then become many different finished goods, which then receive specialized packaging and pass through multiple tiers of distribution, depending on the region of the world. Figure 5 shows an example of a Beauty & Grooming supply chain modeled in Logility's VISO software.

To properly set inventory targets across the supply chain, the supply chain must be modeled at the level of complexity consistent with the production planning system. Therefore, it is modeled as a network in which a stage represents a specific SKU at a location and arcs denote precedence between the stages; this corresponds to SAP's classification for raw materials (ROH), semifinished products (HALB), and finished products (FERT). The mapping must be one-to-one to implement the optimization results.

From a mathematical perspective, the need for multi-echelon inventory optimization is driven by review periods, demand pooling, cost accrual, and network complexity. Although production runs each day, many SKUs are reviewed and replenished weekly or monthly, thus introducing important dependencies between stages. The demand at upstream stages is a complex function of downstream demand, and properly defining how adjacent stages transmit demand

information is necessary. Bossert and Willems (2007) discuss the complexity of review periods.

The costs and demand uncertainties differ significantly at various echelons of the supply chain. On a relative basis, packaged finished goods are significantly more expensive than semifinished goods, which are more expensive than raw materials. Further, because demand is aggregated at fewer locations as it is passed up the supply chain, its relative uncertainty decreases. Combined, these present a tremendous postponement opportunity.

The supply chain in Figure 5 is a relatively simple representation; an average network modeled by Beauty & Grooming has 4,000 to 5,000 stages and 6,000 to 10,000 arcs. Models of this size involve approximately 500 finished goods across the supply chain. Although the supply chain in Figure 5 is significantly more modest in size, it presents a complex acyclic network. The use of common components that range from bottles to chemicals to cardboard, and multiple sources of supply to downstream stages, create a network without any predictable structure. Figure 6(a), which reduces the network from 45 nodes and 52 arcs to 26 nodes and 33 arcs, simplifies Figure 5 to show its network complexity.

Prior to the research we present in the appendix and document in Humair and Willems (2011), the supply chain in Figure 6(a) could not be solved to optimality. Existing exact solution techniques, such as Humair and Willems (2006), require partitioning the network into disjoint bipartite subgraphs; Figure 6(b) shows that this is not possible. Other techniques that rely on linear programming (LP) relaxations require concave stage costs. Because planners require the problem to be solved exactly, these kinds of relaxations were not acceptable to the planning community. In effect, the planners were already using single-echelon tools; they would only adopt new tools that were demonstrably superior.

To demonstrate the value of multi-echelon inventory optimization at P&G, we present the implementation process that began with the North America Cosmetics unit and now spans the entire Beauty business unit, which consists of cosmetics, deodorants, hair care, personal cleansing, prestige fragrances, and skin-care products.

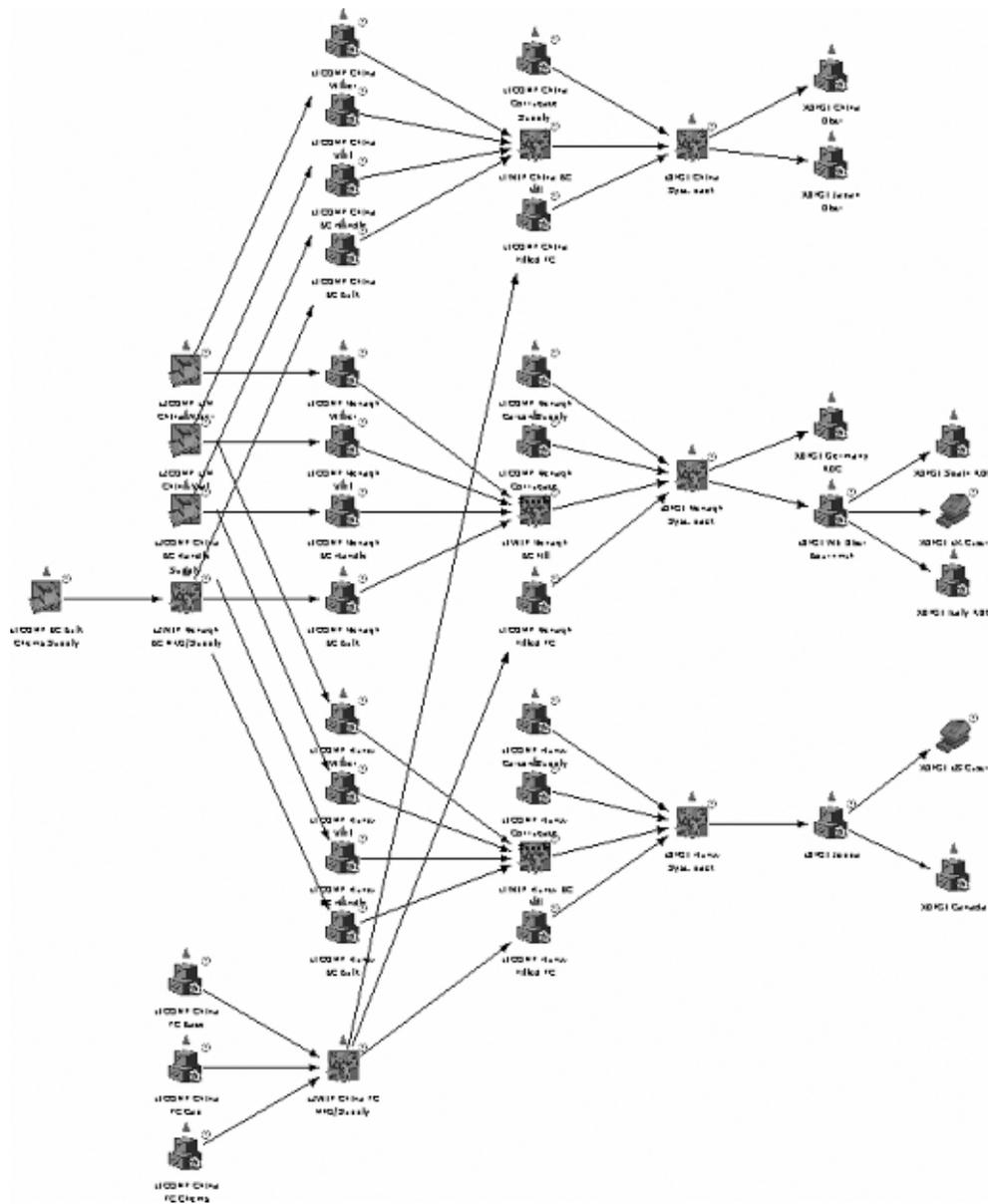


Figure 5: Real-world supply chains at P&G Beauty & Grooming span multiple echelons and have significant component commonality as modeled in this view from Logility's VISO software. *Note:* The text associated with each stage is intentionally blurred to protect confidential information.

Leveraging North America Cosmetics' Success Across the Beauty & Grooming GBU

The Cosmetics supply chain in North America poses a significant challenge to inventory optimization because it supplies the market with over 1,300 standard SKUs and 400–500 promotional SKUs per year.

The supply chain uses over 4,000 unique materials from more than 100 vendors. Despite this complexity, Beauty & Grooming was able to leverage its internal supply chain mastery, coupled with single-stage inventory tools, to reduce its cosmetics inventory by half from 1999 to 2004. Improved capabilities and

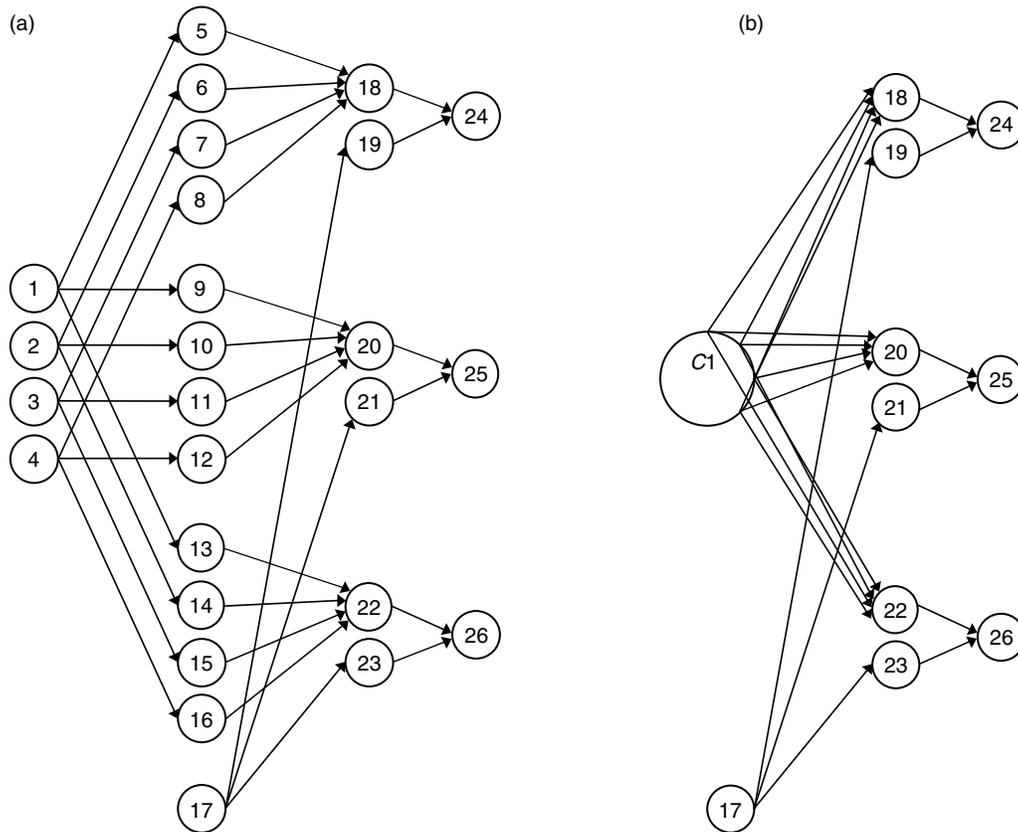


Figure 6: Figure 6(a) simplifies Figure 5 to its underlying acyclic network. Figure 6(b) further reduces 6(a) by representing its largest cluster as a single cluster node, C1.

responsiveness in manufacturing and distribution, real-time information and planning systems, and end-to-end integration from suppliers to retailers all contributed to this success. However, continuing to reduce the leaner inventory became increasingly difficult. Although results reached a plateau in 2005, the business need to continue to free up cash and reduce the risk of obsolescence did not subside. Supply chain leaders decided to replace the single-stage inventory optimization toolset with multiechelon optimization technology.

Figure 7 is a simplified Logility VISO multiechelon model of one product family (liquid makeup) within the total North America cosmetics supply chain. The chain consists of eight unique raw materials (plus another 50 shared with other products that are not shown), 10 blank uncolored work-in-process (WIP) materials, 24 colored WIP materials, 150 pack-

aging materials, 18 intermediate subassemblies (i.e., partially assembled finished goods), and 75 finished goods that move from finished packaging to US and Canada distribution centers, and ultimately to retail customers. Intermediate subassemblies also must satisfy demand for promotional items.

The model incorporates existing service policies (e.g., service level target of 99.5 percent case fill rate), material lead times (generally ranging from 7 days to 8 weeks), production times (1 to 2 days), review periods (7 to 28 days), transportation and movement times (1 to 7 days), quality assurance durations (1 to 5 days), and costs added at each location. Demand characterization (mean and standard deviation) for each finished-goods SKU was based on the previous 13 weeks of actual shipments and forecast, and the future 13 weeks of forecast.

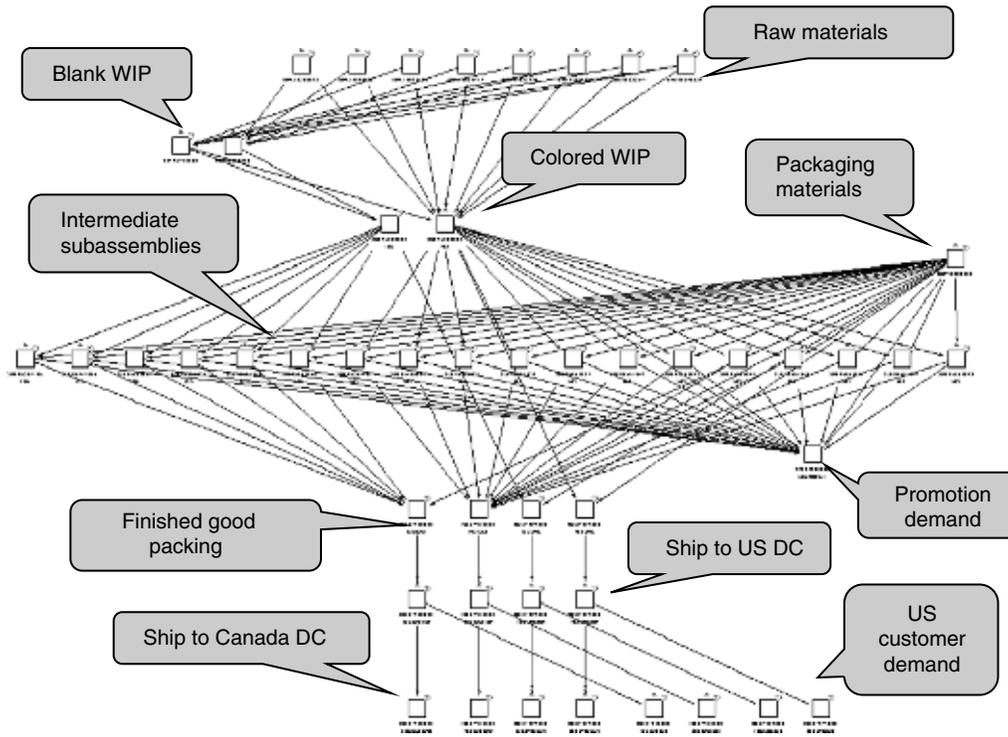


Figure 7: In this screenshot of the multiechelon inventory model for liquid makeup, all 75 finished goods are modeled in a network that has 500 stages and 700 arcs. A stage represents an SKU at a location; an arc captures the bill-of-materials goes-into relationship between SKU locations.

Applying Logility’s multiechelon optimization algorithms to the cosmetics liquid-makeup portion of the supply chain yielded a change in level and placement of inventory safety stocks, while ensuring

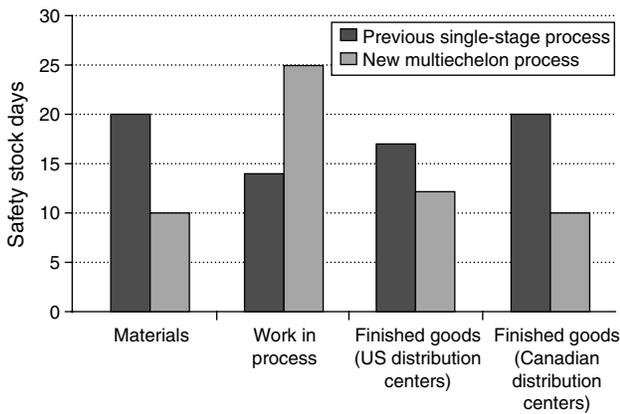


Figure 8: Multiechelon inventory optimization shifted safety stock to the work-in-process echelon, reducing inventory at downstream finished goods locations in the United States and Canada. Raw materials inventory was also reduced.

that the target 99.5 percent service level was protected. Figure 8 provides an overview of the change in safety-stock days on hand across the four major safety-stock inventory echelons—materials, WIP, finished goods in the US distribution center, and finished goods in the Canada distribution center. Safety-stock days decreased in materials and finished goods, and increased in WIP.

Most importantly, Figure 9 provides the safety stocks in dollars. The total investment in safety stock for this supply chain was reduced by 17 percent because the dollar reduction in finished goods and materials far outweighed the dollar increase in work in process.

Application of the multiechelon technology to the entire North America cosmetics supply chain yielded even better results, with the total inventory reduction reaching 7 percent. During the implementation, the cosmetics supply chain team also identified a potential four percentage points of incremental reduction by doing “what-if” analyses of current operating policies.

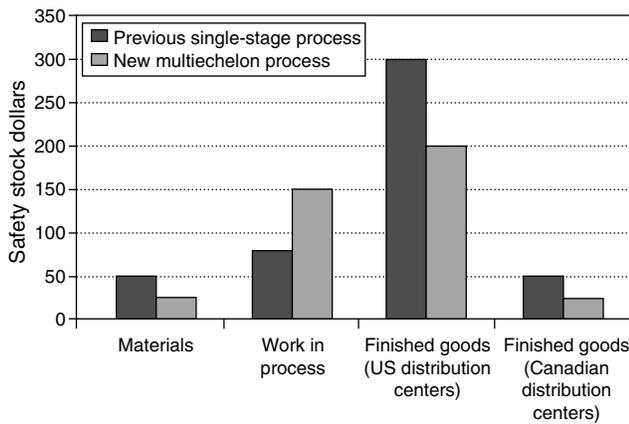


Figure 9: Multiechelon inventory optimization reduced safety stock cost by more than 17 percent, equating to a 5 percent reduction in total inventory cost.

Implementation of the multiechelon model in North America Cosmetics provided 7 percentage points of the 9 percent inventory investment reduction that was achieved from FY 2005–2006 to 2006–2007 (see Figure 10), while maintaining its customer service levels above target; that is, multiechelon inventory technology was credited with 77.8 percent of the inventory savings at North America Cosmetics. In addition, the technology has been a key enabler of subsequent annual reductions of 2–3 percent since the initial implementation. The most significant change was reducing some review periods in finished-goods inventory. The planners could understand the impact

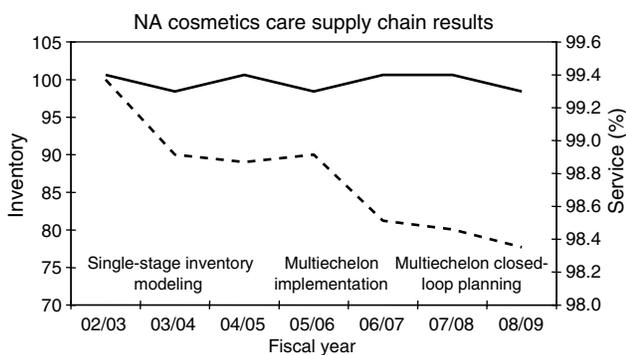


Figure 10: Total inventory in North America Cosmetics decreased 9 percent during the multiechelon implementation process. Multiechelon inventory optimization was credited with 78 percent of this reduction. With the technology established as a backbone of the inventory planning process, sustained reductions of 2–3 percent have been achieved.

of these changes because the supply chain echelons were linked.

It is worth noting that these financial savings were achieved on top of an already strong base of inventory reductions that had been achieved by implementing the single-stage inventory models. These savings were at a site that was a sophisticated consumer of single-stage inventory models—not at a site new to inventory optimization.

As the Logility technology was rolled out beyond P&G Cosmetics to the other Beauty & Grooming supply chains worldwide, a standard multiechelon work process using the P&G standard SAP platform (Figure 11) was developed. The process ensures consistent data visibility and accuracy. Data are extracted from the SAP system modules, passed through an internally developed database for formatting and manipulation, and then passed into the Logility multiechelon software, which automatically maps and ultimately optimizes the model. As part of a monthly workflow, planners review the resulting recommended safety stocks and upload them back into the appropriate SAP systems.

P&G Beauty & Grooming also set up a global support structure (Figure 12) for its multiechelon technology to enable maintenance of current implementations, user skill development, expansion to new supply chains (e.g., acquisitions), expansion of model scope (e.g., into suppliers), and further exploitation of the technology into more strategic applications (e.g., sensitivity and what-if analysis). Key users at each business site own the supply chain model and interface with planners in their supply chain to maintain, apply, and leverage their multiechelon model. Regional leaders in Asia, Latin America, North America, and Europe, who are part of P&G’s Product Supply organization, provide training and support for the site key users, lead new implementations in their regions, identify unique regional requirements, and implement work-process enhancements. A global Product Supply leader supports the regional leaders, identifies global enhancements, maintains work-process standardization across regions, and provides overall strategic direction. These leaders receive IT support from the Supply Network Solutions (SNS) organization, which is within P&G Global Business Services, to maintain hardware and software capabilities.

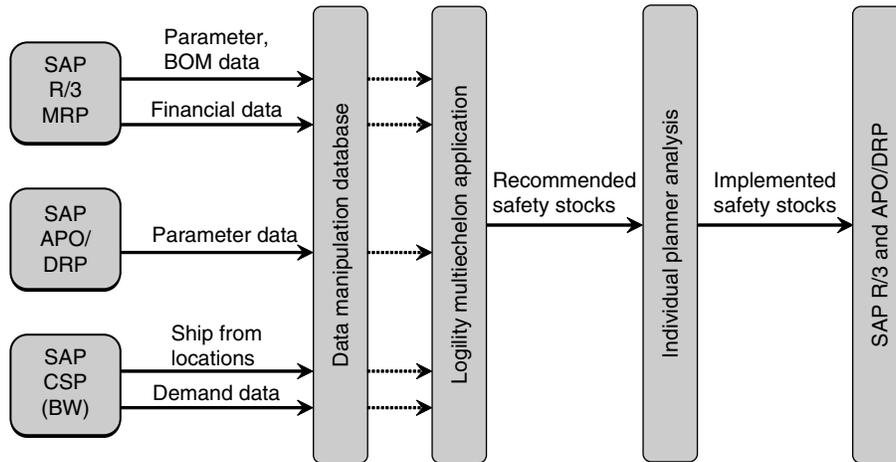


Figure 11: As part of the multiechelon inventory process, P&G implemented a standardized interface between its supply chain related systems and the Logility software. This allows the automatic creation of inventory models and targets for planner approval.

Applying multiechelon technology to the North American Cosmetics supply chain, although one of the most complex in P&G, is only one of 31 successful implementations in P&G’s Beauty & Grooming (see Figure 13). Multiechelon models are in place in every geographic region; associated work processes are executed as frequently as biweekly. As of February

2010, over 85 percent of all SKUs in Beauty & Grooming are being modeled using the multiechelon inventory process. Similar to the North America Cosmetics example described above, these 30 supply chains are achieving total inventory reductions in the 3–7 percent range and are maintaining service levels at or above target.+

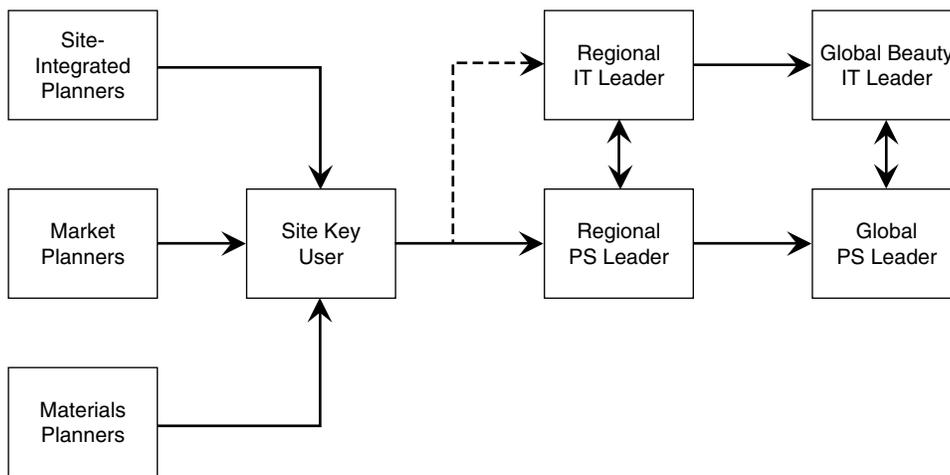


Figure 12: A global workflow structure has been implemented to support multiechelon decision making across Beauty & Grooming.

Region	Number of models	Average number of echelons	Average number of model nodes	Average number of model arcs	Key area of model complexity
North America	10	5	5,000	7,500	SKUs and materials
Asia	8	8	3,500	10,000	Distribution network
Europe	10	6	4,000	6,000	SKUs and materials
Latin America	3	6	2,000	6,500	Distribution network

Figure 13: As of February 2010, 31 multiechelon inventory models are in production in Beauty Care; 85 percent of Beauty & Grooming's SKUs are modeled in the multiechelon inventory process.

Conclusion

By marrying the tools of inventory optimization with the people in P&G's horizontal planning networks, P&G has defined an inventory management process that has significantly reduced its total inventory investment. The work began with single-stage inventory models designed by P&G's OR group and implemented in spreadsheets that planners can use directly. To this day, these spreadsheet tools drive 60 percent of P&G's business. For more complex supply chain networks, multiechelon inventory models have replaced the single-stage models, producing additional average inventory reductions of 7 percent. These multiechelon models now drive 30 percent of P&G's business.

P&G's work on inventory optimization is not standing idle. We assume that another 40 percent of the single-stage models will migrate to multiechelon models in the next three years. Beauty Care has also spearheaded a closed loop of inventory optimization in which tactical inventory models are used to generate more rigorous strategic analysis that can then feed the next-generation design of the supply chain. Furthermore, P&G continues to work in related areas, such as improving the forecast through point-of-sale-based demand sensing.

Appendix. Multiechelon Modeling Framework

The guaranteed service (GS) model of inventory placement forms the underpinnings of the multiechelon inventory optimization (MEIO) tool. Graves and Willems (2003) summarize the specifics of the GS framework and compare it to other MEIO approaches.

Guaranteed Service Model Formulation

In the GS framework, the supply chain is defined as a network with node set N and arc set A . Each node corresponds to a stage in the supply chain. A stage is a processing activity for a given SKU and location in the supply chain, such as manufacturing an item, transporting an item from one location to another, or packaging an item. Arcs denote the precedence relationship between stages. To put this in a system context, if one is using a system (e.g., SAP), stages correspond to raw materials (ROHs), semifinished products (HALBs), and finished products (FERTs). The arcs correspond to the bill-of-materials (BOM) routes that convert ROHs to FERTs.

In its simplest form, the GS model requires very little data. T_j is the lead time at stage j ; this is the time required to complete the processing requirements of stage j , assuming that all its raw materials are available. s_j is the maximum outgoing service time at stage j ; this is the longest time that stage j can take to fulfill demand after it receives an order. There are two decision variables at each stage: incoming service time and outgoing service time. The outgoing service time, S_j , is the delivery time stage j quotes its adjacent downstream stages. The incoming service time, SI_j , is the longest outgoing service time from upstream adjacent stages quoted to stage j .

Each stage has a cost function, $c_j(SI_j, S_j)$, which is a function of its incoming and outgoing service times. The cost function represents the cost of holding inventory at the stage in all its various forms.

The GS model can be formulated as the following mathematical program **P**:

$$\begin{aligned}
 \mathbf{P} \quad & \min \sum_{j=1}^{|N|} c_j(SI_j, S_j) \\
 \text{s.t.} \quad & S_j - SI_j \leq T_j \quad \forall j \in N \\
 & SI_j - S_i \geq 0 \quad \forall (i, j) \in A \\
 & S_j \leq s_j \quad \forall j: \nexists k \in N \mid (j, k) \in A \\
 & S_j, SI_j \geq 0, \text{ integral} \quad \forall j \in N.
 \end{aligned}$$

The objective function minimizes the total stage cost. The first constraint imposes a rational operating constraint on a stage's outgoing service time: a stage's outgoing service time cannot exceed the longest service time quoted to the stage plus its own lead time.

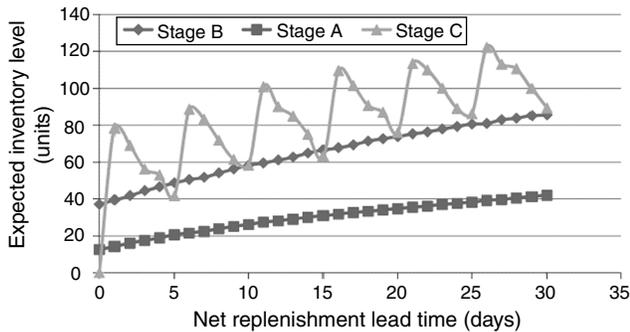


Figure A.1: Generalized stage cost functions occur in the presence of advanced supply chain dynamics (e.g., review periods).

The second constraint defines the incoming service time at a stage to be at least as large as the longest outgoing service time quoted to the stage. The third constraint enforces an upper bound on a demand stage's outgoing service time; the final constraints require service times to be nonnegative and integer.

More complex forms of the GS model capture advanced supply chain dynamics, such as nonnested review periods (Bossert and Willems 2007) or nonstationary demand (Neale and Willems 2009). These advanced dynamics require more data to fully specify the model; however, the general structure is similar in form to **P**.

Novelty of Solution Approach

If the model incorporates dynamics such as review periods or batching, the stage cost function is neither concave nor monotone. Figure A.1 illustrates why standard GS optimization approaches cannot be used for generalized cost functions, because they assume either an extreme-point property of the optimal solution or piecewise linearity of the cost function. This figure also demonstrates why standard nonconvex optimization approaches fail—because of their assumptions about the structure of the objective function (e.g., submodularity or quasi concavity). Search algorithms also typically either require an objective function structure (e.g., conjugate gradient methods) or use an unstructured strategy (e.g., genetic algorithms) and are usually less efficient.

Therefore, we based our approach to constructing an algorithm on two simple observations: first, that the spanning tree algorithm from Graves and Willems (2000) could handle these complicated cost structures elegantly with a minor reformulation; second, that the solution to a well-chosen spanning tree relaxation of a general network was very often (when we observed it qualitatively) close to the true optimal solution. The problem was how to move from the relaxed solution to the network-optimal solution.

The answer was not obvious, because the spanning tree algorithm cannot be used to get an optimal solution by bounding a single variable, as Humair and Willems (2011) discuss. By observing the existence of another relationship between the inbound service times and network partitioning, we were able to construct an algorithm that could move efficiently from the relaxed solutions to the network-optimal solutions. The efficiency of the resulting algorithm was surprising, and its attractive property was its provable optimality. Humair and Willems (2011) provide details.

References

- Bossert, J. M., S. P. Willems. 2007. A periodic-review modeling approach for guaranteed service supply chains. *Interfaces* 37(5) 420–435.
- Farasyn, I., K. Perkoz, W. Van de Velde. 2008. Spreadsheet models for inventory target setting at Procter & Gamble. *Interfaces* 38(4) 241–250.
- Graves, S. C., S. P. Willems. 2000. Optimizing strategic safety stock placement in supply chains. *Manufacturing Service Oper. Management* 2(1) 68–83.
- Graves, S. C., S. P. Willems. 2003. Supply chain design: Safety stock placement and supply chain configuration. A. G. de Kok, S. C. Graves, eds. *Supply Chain Management: Design, Coordination and Operation. Handbooks in Operations Research and Management Science*, Vol. 11. North-Holland Publishing Company, Amsterdam, 95–132.
- Humair, S., S. P. Willems. 2006. Optimizing strategic safety stock placement in supply chains with clusters of commonality. *Oper. Res.* 54(4) 725–742.
- Humair, S., S. P. Willems. 2011. Optimizing strategic safety stock placement in general acyclic networks. *Oper. Res.* Forthcoming.
- Murphy, R. A. 1975. Inventory control with gamma demand and gamma lead times. *Internat. J. Systems Sci.* 6(1) 81–85.
- Neale, J. J., S. P. Willems. 2009. Managing inventory in supply chains with nonstationary demand. *Interfaces* 39(5) 388–399.