
Volume 236, Issue 2, 16 July 2014
ISSN 0377-2217
ELSEVIER
EUROPEAN JOURNAL OF OPERATIONAL RESEARCH
Available online at www.sciencedirect.com
ScienceDirect
www.elsevier.com/locate/ejor

European Journal of Operational Research 236 (2014) 522–533

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Production, Manufacturing and Logistics

Modeling sourcing strategies to mitigate part obsolescence

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ARTICLE INFO

Article history:
Received 14 December 2012
Accepted 13 January 2014
Available online 23 January 2014

Keywords:
Manufacturing
Part obsolescence
Life-of-type buy
Part substitution
Line redesign
Diminishing manufacturing sources and material shortage

ABSTRACT

Part obsolescence is a common problem across industries, from avionics and military sectors to most original equipment manufacturers serving industrial markets. When a part supplier announces that a part will become obsolete, the OEM can choose from a number of sourcing options. In practice, the three most commonly adopted mitigation strategies are: (1) a lifetime, or life-of-type (LOT), buy from the original supplier; (2) part substitution, which finds a suitable alternative; and (3) line redesign, which modifies the production line to accommodate a new part. We first develop a framework incorporating fixed cost, variable cost, leadtime, demand uncertainty and the discount rate to directly compare and characterize these three sourcing strategies in a static context. We next formulate an integrated sourcing approach that starts with a bridge buy and may continue with part substitution or line redesign when the originals parts are depleted. Through numerical studies, we identify the joint impact of the problem parameters on the static and integrated sourcing strategies and the optimal choice among them. While the integrated sourcing approach outperforms the static ones in many cases it is not a dominant strategy.

1. Introduction

Manufacturers in the avionics and military industries have long had to deal with the challenge of effectively managing part obsolescence. In these industries, product lifecycles are measured in decades so effectively managing the parts supply base is a necessary condition for survival. Furthermore, this issue is significant enough that the US military defines this problem area by the acronym DMSMS: diminishing manufacturing sources and material shortages. Over the past decade, the problem of part obsolescence has become a significant issue for original equipment manufacturers (OEMs) in industrial sectors like communications, construction equipment, medical devices, and transportation (Souza, 2003, Shotter, 2012). The increasing importance of part obsolescence in the industrial sector can be directly attributed to the sector's increasing reliance on parts developed for the consumer-products market. In the consumer-products market, parts evolve rapidly and require frequent upgrading. As a consequence, the long life of the industrial products and the shorter life of its parts create a lifecycle-mismatch problem.

Saal and Dube (2001) report that the average part lifecycle has decreased from 7 to 10 years in the early 1990s to 2 to 5 years in 2001. In some industries like wireless chips the lifecycles of parts can be less than one year. There are thousands of components that reach their end of life (EOL) every month as suppliers prune their offerings (Tyler, 2004), and distributors like Avnet report that they are being asked by customers more than 20 times a week to solve EOL situations (Sullivan & Lamb, 2002). An industry has sprung up that only manufactures parts that have been declared obsolete by the original part manufacturer; one such example is Rochester Electronics (2009). Part obsolescence has been becoming an increasingly acute problem for many equipment companies (Shotter, 2012) because of significant losses associated with these events.

When a part supplier makes a part obsolescence announcement, the supplier notifies the OEM about the impending obsolescence event in advance, typically six months ahead of the end of production. Livingston (2000) documents the set of mitigation strategies OEMs have once they are notified of such an event: (1) a lifetime buy, also referred to as a life-of-type (LOT) buy, in which the OEM places its final order from the original supplier to satisfy future demand assuming it will take no other actions; (2) a bridge buy, which is a last-time buy from the original supplier that is then followed by another mitigation tactic; (3) a part substitution, by finding a part with equivalent form, fit, and functions to the obsolete part; (4) an alternate supplier, by finding a vendor supplying the same part; (5) an aftermarket solution, in which a distributor or a manufacturer takes over the production line or inventory from the original supplier; (6) a line redesign/design refresh, in which the OEM needs to design out the original part and replace it with a new part; (7) an emulation, which uses a new-technology component to emulate

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http://dx.doi.org/10.1016/j.ejor.2014.01.025
0377-2217/

the original component's functionality; (8) uprating, which uprates the parts to adapt to a broader environment; and (9) a reclaim from salvaged parts, which is usually the last choice for the OEMs. While there are engineering differences among the aforementioned solutions, operationally some share similarities with others. For example, LOT buy and bridge buy only differ if there is a subsequent action. Emulation and uprating are similar to line redesign or design refresh, which all require substantial engineering efforts. Alternative and aftermarket solutions are similar to part substitution, which all require identifying new suppliers.

Stogdill (1999) and Livingston (2000) report that in practice the aforementioned solutions occur with the following frequencies: (1) part substitution (67%); (2) LOT buy (20%); (3) line redesign with a bridge buy (12%); and (4) emulation (1%). The remaining strategies are adopted less frequently. In the case of LOT buy, a relationship already exists with the supplier so only an ordering process is needed. For part substitution, the OEM must engage in a search process to identify a supplier. While the Internet has significantly reduced this effort (Saal & Dube, 2001), the OEM still must test the form, fit, and functions of the part for the product; this incurs cost and takes time. Line redesign is an even more complicated process. It requires evaluating the part to be designed out, finding a new part to be inserted into the new assembly line, and finally testing the line and product. These steps require significant capital, engineering resources, and calendar time. To focus on the salient operational differences, but not the technological and engineering aspects, we evaluate LOT buy, part substitution and line redesign because they represent the three most frequently adopted part-obsolescence mitigation strategies.

These three mitigation strategies can be considered in a static or integrated context. In the static context, a single mitigation strategy must be chosen after the obsolescence announcement and executed without modification. We construct a unified framework to choose the optimal strategy in this static setting. We next develop an integrated sourcing approach which starts with a bridge buy followed by a recourse action of part substitution, line redesign, or no action. Such an integrated strategy will be shown to be a better solution under certain, but not all, conditions. While existing literature studies the LOT buy problem, other part-obsolescence solutions have not been analytically modeled, nor have these approaches been compared in a unified framework. Focusing on sourcing a product subject to part obsolescence, we position our work at the intersection of supply management and product lifecycle management.

Following the introduction, we review the relevant literature in Section 2. In Section 3, we model the three static sourcing strategies, and then present some structural results for the derived models. In Section 4, we propose an integrated sourcing approach that combines the three static sourcing options with a recourse option. In Section 5, we numerically study the optimal sourcing strategies. Section 6 concludes the paper and presents some future research directions.

2. Literature review

Sandborn and Myers (2008) define part obsolescence in the domain of sustainable engineering, and review the technological and operational aspects for part obsolescence. The existing work relevant to the management of part obsolescence includes: (1) part lifecycle characterization, (2) part obsolescence forecasting, (3) product deletion, and (4) lifecycle planning. Lifecycle characterization and obsolescence forecasting are reviewed by Singh, Sandborn, Geiser, and Lorenson (2004), and product deletion is reviewed by Avlonitis, Hart, and Tzokas (2000). Forecasting the time of parts obsolescence is itself an issue (Sandborn, Prabhakar, & Ahmad, 2011) if the solution to part obsolescence is proactive and particularly if there are multiple parts becoming obsolete. If the obsolescence timing is uncertain, Kumar and Saranga (2010) propose bandit models to identify the best mitigation strategies for part obsolescence. The part-obsolescence solutions in our paper are reactive in the sense that all the solutions are planned after a part is announced obsolete. Although proactive solutions to part obsolescence are urged (Bradley & Guerrero, 2008) in the product design stage, reactive solutions are still the dominant ones seen in practice. Recently, Rojo, Roy, and Shehab (2009) have compiled a long list of papers in this field.

In a pioneering work at Boeing, Porter (1998) defines the basic decision for part obsolescence in the avionics industry as whether to make a LOT buy of parts to last through the product's lifecycle or to initiate a line redesign that generates a new part to replace the old one. By assuming deterministic demand, Porter developed some simple rules of thumb to allow planners to make this tradeoff taking into account inventory cost, financial cost, and the fixed cost associated with line redesign. In avionics, redesign can be very expensive, so it is only explored after the other choices are exhausted. Using Monte Carlo simulation, Feng, Singh, and Sandborn (2007) conduct a case study on part obsolescence of Motorola's telecommunication base equipment. They find that Motorola had intended to over-stock the parts purchased by a LOT buy or bridge buy because too little emphasis was placed on the cost of inventory and the time value of money. Singh and Sandborn (2006) also design a simulation tool that calculates cost associated with multiple planned line redesigns to test whether line redesign is superior to other mitigation strategies. Porter (1998), Singh and Sandborn (2006), and Feng et al. (2007) all discuss line redesign or design refresh solutions in the presence of multi-component obsolescence, where exact modeling is often impossible. To generate managerial insights, Cattani and Souza (2003) study the case in which the supplier announces the intention to discontinue producing a part but the end-of-life event can be delayed, and Bradley and Guerrero (2009) build two-period LOT-buy models if two parts obsolete sequentially in the two periods. To embed different sourcing strategies in a product lifecycle profile, we assume that there is only one part that is subject to part obsolescence and the timing of this obsolescence event is known.

A problem related to part obsolescence for long-life industrial products is the end-of-life (EOL) service problem on spare parts for short-life consumer electronics, where a manufacturer is contracted to provide maintenance service to a customer even after production has ceased. In this case, the manufacturer has to place the final orders on some spare parts which might be discontinued from the part suppliers, using a forecast for future part failures. A number of papers, Teunter and Fortuin (1998, 1999), Spengler and Schroter (2003), van Kooten and Tan (2009), Teunter and Haneveld (1998), Sahyouni, Savaskan, and Daskin (2009), Pourakbar and Dekker (2012), Pourakbar, Frenk, and Dekker (2012) and van der Heijden and Iskandar (2013) have addressed this problem under various assumptions where the final order is the primary decision but additional remedy is possible. EOL service could be regarded as a sub-problem to the part obsolescence problem.

Embedding supply management into product lifecycle management has been practiced in industry. Shen and Willems (2012) study sourcing a short lifecycle product with parts possessing different operational properties. In contrast, this paper studies sourcing a long lifecycle product subject to part obsolescence by one or more mitigation strategies over time.

3. Static sourcing strategies

Part-obsolescence solutions are typically implemented in a static fashion because it is easy to manage and, by construction, the part is already near its end of life. In this section we model and compare the three most frequently adopted static sourcing strategies.

3.1. Model preliminaries

To model the sourcing options, we first discretize the planning horizon by index i, $i = 1, 2 \ldots, N$, where $i = 1$ refers to the time at which the sourcing decision is made. This occurs in the time window after it is announced that the original part will become obsolete. Likewise, $i = N$ refers to the end of the product lifecycle, which is assumed to be known. The demand in period i is assumed to be a random variable, X_i, which is characterized at the beginning of the planning horizon. In each period, we first observe demand, receive any arriving order if there is one, and then fulfill demand from available parts.

We use a multivariate function $f(X_1 \cdots X_N)$ to denote the joint demand distribution for periods 1 to N. The mean demand in period i and the joint demand distributions for periods 1 to i are calculated as: $\mu_i = \int \cdots \int f(X_1 \cdots X_i) X_i \, dX_1 \cdots dX_i, f(X_1 \ldots X_i) = \int \cdots \int f(X_1 \cdots X_N) \, dX_{i+1} \cdots dX_N$.

We further define the cumulative demand $Y_{1,i} = \sum_{j=1}^{i} X_j$ for each $i = 1$ to N, with the probability distribution as $f_{Y_{1,i}}(x) = \int \cdots \int f(X_1 \cdots X_i) \delta\left(x - \sum_{j=1}^{i} X_j\right) dX_1 \cdots dX_i$ where $\delta(\bullet)$ is the impulse function satisfying $\int_{-\infty}^{\infty} \delta(x)\, dx = 1,\ \delta(0) = \infty$. The cumulative distribution function for Y_{1i} is then defined by $F_{Y_{1,i}}(x) = \int_0^x f_{Y_{1,i}}(y)\, dy$.

We begin by presenting three static models corresponding to the three strategies most frequently employed in practice: LOT buy, part substitution, and line redesign. Each model is static in the sense that a sole sourcing strategy is adopted in period 1 and continued over the entire N periods.

To formulate the models, we introduce a set of financial parameters:

- k_o the fixed cost of LOT buy
- c_o the per-unit variable cost associated with LOT buy
- h the per-unit inventory holding cost per period of the original part
- b the per-unit penalty cost per period of the original part
- v the per-unit salvage value of the original part
- k_s the fixed cost associated with part substitution
- c_s the per-unit variable cost associated with part substitution
- k_d the fixed cost associated with line redesign
- c_d the per-unit variable cost associated with line redesign
- p_i the unit selling price for the end product in period i, with the cost of all parts other than the obsolete part netted out
- α the financial discount factor per period

We formulate the profit-maximization variant of each problem but it is easy to convert each formulation to its equivalent cost-minimization problem.

We further have:

Assumption 1.

(a) $p_1 \geqslant \cdots \geqslant p_N > c_s,\quad p_N > c_d,\quad p_N > c_o$; (b) $k_d > k_s,\quad k_d > k_o$; (c) $c_d \geqslant c_o > v,\quad c_s \geqslant c_o > v$

Assumption 1(a) assumes that the prices will not increase during the product lifecycle which is the case in most situations; in addition, the prices are assumed higher than any part's cost. The fixed cost in each sourcing option is comprised of non-recurring cost such as testing cost and the normal ordering cost. 1(b) reflects the fact that line redesign incurs significant non-recurring cost (NRC) (Wilson, 2005) while there is no such cost associated with LOT buy and it is much less for part substitution. Even though LOT buy does not incur NRC, its order processing cost can be relatively large. We do not impose a relation between the fixed cost of LOT buy and that of part substitution. The assumptions on the fixed costs align with reality in the avionics and military sectors (Livingston, 2000). For 1(c), it is reasonable that the variable cost of the original part is less than the variable costs associated with either part substitution or line redesign as the original part is being phased out. We address the relation between c_d and c_s later.

3.2. LOT buy

In this sourcing approach, we first assume:

Assumption 2. *The delivery of the LOT buy or bridge buy is immediate, or the leadtime is zero.*

The leadtime for the original part is assumed zero because in comparison to other sourcing approaches, there is no engineering effort or process change required to execute the order.

If there is a positive LOT buy quantity,Q_1, for a realized demand profile of $\{X_1 \cdots X_N\}$, the sales in period i ($i = 1, 2, \ldots, N$) are then $\min\left(X_i, \left(Q_1 - \sum_{j=1}^{i-1} X_j\right)^+\right)$ because the inventory is depleted as much as possible under non-increasing prices and inventory holding cost. The inventory of parts at the end of period i is $\left(Q_1 - \sum_{j=1}^{i} X_j\right)^+$ while lost sales are $\min\left(X_i, \left(\sum_{j=1}^{i} X_j - Q_1\right)^+\right)$. We assume the initial inventory of the original parts as zero. The LOT buy problem is then to maximize profit:

Problem LOT

$$\begin{aligned} Max_{Q_1>0} \Pi_{LOT}(Q_1) = &-k_o - c_o Q_1 \\ &+ E_{X_1 \ldots X_N} \left\{ \sum_{i=1}^{N} \alpha^{i-1} \left[p_i \min\left(X_i, \left(Q_1 - \sum_{j=1}^{i-1} X_j\right)^+\right) \right.\right. \\ &\left. - b \min\left(X_i, \left(\sum_{j=1}^{i} X_j - Q_1\right)^+\right) \right] \\ &\left. - h \sum_{i=1}^{N-1} \alpha^{N-1} \left(Q_1 - \sum_{j=1}^{i} X_j\right)^+ + \alpha^{N-1} v \left(Q_1 - \sum_{j=1}^{N} X_j\right)^+ \right\} \end{aligned}$$

Allowing purchasing quantities to be non-integer, the above result can be further simplified as:

$$\begin{aligned} Max_{Q_1>0} \Pi_{LOT}(Q_1) = &-k_o - b \sum_{i=1}^{N} \alpha^{i-1} \mu_i + (p_1 + b - c_o) Q_1 \\ &- \sum_{i=1}^{N-1} \alpha^{i-1} [p_i + b(1-\alpha) + h - \alpha p_{i+1}] \\ &\times \int_0^{Q_1} F_{Y_{1,i}}(x)\, dx - \alpha^{N-1}(p_N + b - v) \\ &\times \int_0^{Q_1} F_{Y_{1,N}}(x)\, dx \end{aligned} \quad (1)$$

By adding the fixed cost and penalty cost, Problem LOT, in (1), has enhanced the model in Cattani and Souza (2003). In addition, we have provided a new derivation for the formulation of the LOT problem. We further rewrite the above objective function as

$$\Pi_{LOT}(Q_1) = -k_o - b \sum_{i=1}^{N} \alpha^{i-1} \mu_i + \Pi_{LOT,0}(Q_1) \quad (2)$$

$$\Pi_{LOT,0}(Q_1) = (p_1 + b - c_o)Q_1 - \sum_{i=1}^{N-1}\alpha^{i-1}[p_i + b(1-\alpha) + h - \alpha p_{i+1}]\int_0^{Q_1} F_{Y_{1,i}}(x)\,dx - \alpha^{N-1}(p_N + b - v) \times \int_0^{Q_1} F_{Y_{1,N}}(x)\,dx \quad (3)$$

We have:

Lemma 1. *$\Pi_{LOT}(Q_1)$ is concave if the order quantity is non-zero. The optimal LOT-buy quantity, Q_1^*, satisfies*

$$(p_1 + b - c_o) - \sum_{i=1}^{N-1}\alpha^{i-1}[p_i + b(1-\alpha) + h - \alpha p_{i+1}]F_{Y_{1,i}}(Q_1^*) - \alpha^{N-1}(p_N + b - v)F_{Y_{1,N}}(Q_1^*) = 0 \quad (4)$$

All the proofs are in Appendices A–H. *It is clear that* (4) *generalizes the one-period newsvendor solution to the multi-period scenario by considering the discount factor, inventory holding cost and price difference across periods. The role of the fixed cost will be discussed in* Section 3.5.

3.3. Part substitution

In general, part substitution begins with a bridge buy from the original supplier while simultaneously identifying the substitute part. The substitute part is assumed available in period L_1, where L_1 is the leadtime for the substitute part. We fulfill demand using the original part, purchased from the bridge buy, as long as it is available and otherwise use the substitute part from period L_1 onwards. To simplify the exposition, we start with:

Assumption 3. *The leadtime to use the substitute part is assumed to be* $L_1 = 1$.

This assumption allows the search process for the substitute part to be quick enough to enable its adoption in the first period. In reality, OEMs often maintain lists of substitute parts. The relatively low cost of the original part might make a bridge buy optimal even if the substitute part is available immediately. We first focus on the conditions under Assumption 3 in order to generate some managerial insights but will discuss the case that $L_1 > 1$, corresponding to the case when the search process for the substitute parts takes significant time.

We denote Q_2 as the bridge-buy quantity while Assumption 2 applies. For a realized demand profile of $\{X_1 \cdots X_N\}$, the stock of the original parts will be depleted as in the LOT buy; if this stock runs out in period i then we use the substitute parts in the amount of $X_i - \min\left(\left(Q_2 - \sum_{j=1}^{i-1}X_j\right)^+, X_i\right)$. The expected profit of the system is calculated by summing over all the possible profiles of the demand. We then solve.

Problem SUB-BB

$$\text{Max}_{Q_2 > 0}\Pi_{SUB-BB}(Q_2) = -k_o - k_s - c_oQ_2 + E_{X_1\cdots X_N}\left\{\sum_{i=1}^{N}\alpha^{i-1}\left[p_iX_i - c_s\left(X_i - \min\left(\left(Q_2 - \sum_{j=1}^{i-1}X_j\right)^+, X_i\right)\right)\right] - h\sum_{i=1}^{N-1}\alpha^{i-1}\left(Q_2 - \sum_{j=1}^{i}X_j\right)^+ + \alpha^{N-1}v\left(Q_2 - \sum_{j=1}^{N}X_j\right)^+\right\}$$

Taking the expected value over demand, we have

$$\Pi_{SUB-BB}(Q_2) = -k_o - k_s + \sum_{i=1}^{N}\alpha^{i-1}(p_i - c_s)\mu_i + (c_s - c_o)Q_2 - [h + (1-\alpha)c_s]\sum_{i=1}^{N-1}\alpha^{i-1}\int_0^{Q_2} F_{Y_{1,i}}(x)\,dx - \alpha^{N-1}(c_s - v)\int_0^{Q_2} F_{Y_{1,N}}(x)\,dx \quad (5)$$

We further recast (5) into

$$\Pi_{SUB-BB}(Q_2) = -k_o - k_s + \sum_{i=1}^{N}\alpha^{i-1}(p_i - c_s)\mu_i + \Pi_{SUB-BB,0}(Q_2) \quad (6)$$

$$\Pi_{SUB-BB,0}(Q_2) = (c_s - c_o)Q_2 - [h + (1-\alpha)c_s]\sum_{i=1}^{N-1}\alpha^{i-1}\int_0^{Q_2} F_{Y_{1,i}}(x)dx - \alpha^{N-1}(c_s - v)\int_0^{Q_2} F_{Y_{1,N}}(x)\,dx \quad (7)$$

In (7), $\Pi_{SUB-BB,0}(Q_2)$ is the profit generated from fulfilling the demand with a bridge buy of Q_2.

A noteworthy special case is to make no bridge buy of original parts and fulfill demand with only substitute parts. From Assumption 3, the expected profit for this special case is $-k_s + \sum_{i=1}^{N}\alpha^{i-1}(p_i - c_s)\mu_i$ as all the demand can be satisfied. We refer to the general case of part substitution, as defined by problem SUB-BB, as "mixed substitution" and the special case with a zero bridge-buy order as "pure substitution". Now we are able to show:

Lemma 2.

(1) *For the mixed substitution case, the unique, optimal bridge-buy quantity, Q_2^*, is decided by*

$$(c_s - c_o) - [h + (1-\alpha)c_s]\sum_{i=1}^{N-1}\alpha^{i-1}F_{Y_{1,i}}(Q_2^*) - \alpha^{N-1}(c_s - v)F_{Y_{1,N}}(Q_2^*) = 0 \quad (8)$$

(2) *There are two asymptotic observations for c_s:*
  (a) *If $c_s = c_o$, then $Q_2^* = 0$ and the mixed substitution case reduces to pure substitution.*
  (b) *If $p_1 = \cdots = p_N = c_s$, $b = 0$, then the mixed substitution reduces to the LOT buy; namely, $Q_2^* = Q_1^*$ and no substitute part is used since it yields zero profitability.*

Besides the fixed costs, mixed substitution shares similar mathematical structure to LOT buy; therefore, part (1) in Lemma 2 is similar to Lemma 1. Part (2) reflects two asymptotic conditions in which mixed substitution reduces to either the LOT buy or pure substitution. Since these conditions are based on the variable costs regardless of the fixed costs and demand information, we will not further mention these specific situations.

3.4. Line redesign

Inspired by the rule-of-thumb results in Porter (1998), we provide a rigorous formulation for the line redesign approach. Assuming the redesign leadtime as L_2, we define the redesign process either by period n, which reflects when the newly designed part is ready for use, or by period $n - L_2$, which reflects when the line starts to be redesigned. Line redesign might not start immediately at the beginning of the planning horizon because of the cost trade-off between stocking more of the original parts versus utilizing the new parts earlier. In addition, line redesign normally needs much more engineering effort than part substitution, so we have:

Assumption 4. *The leadtime of line redesign, L_2, satisfies $L_2 > L_1$.*

Denoting the bridge-buy quantity as Q_3, the original parts are depleted as in LOT buy. For a realized demand profile of $\{X_1 \cdots X_N\}$, there might be part shortage of $\min\left(X_i, \left(\sum_{j=1}^{i} X_j - Q_3\right)^+\right)$, for each period of $i = 1 \cdots n-1$. From period n onwards, we fulfill the demand in period i with the new parts in the amount of $\left[X_i - \left(Q_3 - \sum_{j=1}^{i-1} X_j\right)^+\right]^+$ if the original parts are already consumed.

In this static setting, the timing of line redesign, once decided, will not be altered later in time. The line redesign problem is to solve:
Problem DS-BB

$$\begin{aligned}
&\mathrm{Max}_{Q_3>0, n\geqslant L_2}\, \Pi_{DS-BB}(Q_3,n) = -k_o - k_d\alpha^{n-1} - c_o Q_3 \\
&+E_{X_1\cdots X_N}\left\{\sum_{i=1}^{N}\alpha^{i-1}p_i \min\left(X_i, \left(Q_3 - \sum_{j=1}^{i-1}X_j\right)^+\right) - h\sum_{i=1}^{N-1}\alpha^{i-1}\left(Q_3 - \sum_{j=1}^{i}X_j\right)^+\right. \\
&-b\sum_{i=1}^{n-1}\alpha^{i-1}\min\left(X_i, \left(\sum_{j=1}^{i}X_j - Q_3\right)^+\right) + \sum_{i=n}^{N}\alpha^{i-1}(p_i - c_d)\left[X_i - \left(Q_3 - \sum_{j=1}^{i-1}X_j\right)^+\right]^+ \\
&\left.+\alpha^{N-1}v\left(Q_3 - \sum_{j=1}^{N}X_j\right)^+\right\}
\end{aligned}$$

where the fixed cost of line redesign, k_d, is discounted in period n. We can simplify and decompose $\Pi_{DS-BB}(Q_3,n)$ by $\Pi_{DS-BB}(Q_3,n) = \Pi_{DS-BB,0}(n) + \Pi_{DS-BB,1}(Q_3,n)$ where

$$\Pi_{DS-BB,0}(n) = -k_o - k_d\alpha^{n-1} - b\sum_{i=1}^{n-1}\alpha^{i-1}\mu_i + \sum_{i=n}^{N}\alpha^{i-1}(p_i - c_d)\mu_i \tag{9}$$

$$\begin{aligned}
\Pi_{DS-BB,1}(Q_3,n) = {} & (p_1 + b - c_o)Q_3 \\
& - \sum_{i=1}^{n-2}\alpha^{i-1}[p_i + h + b(1-\alpha) - \alpha p_{i+1}] \\
& \times \int_0^{Q_3} F_{Y_{1,i}}(x)dx - \alpha^{n-2}(p_{n-1} + h + b - \alpha c_d) \\
& \times \int_0^{Q_3} F_{Y_{1,n-1}}(x)dx - \sum_{i=n}^{N-1}\alpha^{i-1}[h + (1-\alpha)c_d] \\
& \times \int_0^{Q_3} F_{Y_{1,i}}(x)dx - \alpha^{N-1}(c_d - v)\int_0^{Q_3} F_{Y_{1,N}}(x)dx
\end{aligned} \tag{10}$$

From the decomposition of (9) and (10), the solution to Problem DS-BB is stated as:

Lemma 3.

(1) *$\Pi_{DS-BB,1}(Q_3,n)$ is concave in Q_3 for any n. It has a finite optimal solution, $Q^*_{3,n}$, satisfying*

$$\begin{aligned}
0 = {} & (p_1 + b - c_o) - \sum_{i=1}^{n-2}\alpha^{i-1}[p_i + h + b(1-\alpha) \\
& - \alpha p_{i+1}]F_{Y_{1,i}}\left(Q^*_{3,n}\right) - \alpha^{N-1}(c_d - v)F_{Y_{1,N}}\left(Q^*_{3,n}\right) \\
& - \alpha^{n-2}(p_{n-1} + h + b - \alpha c_d)F_{Y_{1,n-1}}\left(Q^*_{3,n}\right) \\
& - \sum_{i=n}^{N-1}\alpha^{i-1}[h + (1-\alpha)c_d]F_{Y_{1,i}}\left(Q^*_{3,n}\right)
\end{aligned} \tag{11}$$

(2) *The optimal timing of line redesign, denoted as n^*, is derived by the following line search:*

$$n^* = \mathrm{Argmax}_{n\geqslant L_2}\left[\Pi_{DS-BB,0}(n) + \Pi_{DS-BB,1}\left(Q^*_{3,n}, n\right)\right]$$

The optimal LOT buy quantity is denoted as $Q^*_3 = Q^*_{3,n^*}$.
It is clear that the fixed costs of the bridge buy and line redesign establish threshold conditions when line redesign will be effective.

3.5. Structural results

Based on the initial solutions to the static models, we now derive some structural results for the three sourcing strategies. This section provides some basic properties for the problem parameters with regard to a single sourcing option and some bilateral relations among the three sourcing strategies.

When both the LOT buy and the bridge orders are non-zero in each of the three sourcing strategies, we have the following comparative static properties:

Proposition 1.

(1) $\frac{\partial Q_1^*}{\partial h} < 0, \quad \frac{\partial \Pi_{LOT}(Q_1^*)}{\partial h} < 0; \quad \frac{\partial Q_1^*}{\partial b} > 0, \quad \frac{\partial \Pi_{LOT}(Q_1^*)}{\partial b} < 0; \quad \frac{\partial \Pi_{LOT,0}(Q_1^*)}{\partial b} > 0$

(2) $\frac{\partial Q_2^*}{\partial h} < 0, \quad \frac{\partial \Pi_{SUB-BB}(Q_2^*)}{\partial h} < 0; \quad \frac{\partial Q_2^*}{\partial c_s} > 0, \frac{\partial \Pi_{SUB-BB}(Q_2^*)}{\partial c_s} < 0$

(3) $\frac{\partial Q^*_{3,n}}{\partial h} < 0, \quad \frac{\partial \Pi_{DS-BB}\left(Q^*_{3,n}\right)}{\partial h} < 0; \quad \frac{\partial Q^*_{3,n}}{\partial b} > 0, \quad \frac{\partial \Pi_{DS-BB}\left(Q^*_{3,n}\right)}{\partial b} < 0$

Most of the above results are intuitive, so we only present proofs for the less obvious ones in Appendix D. *It is clear that (4). Part of this proposition also facilitates the proof for* Proposition 2.

In Section 5 we numerically investigate the joint effects of these problem parameters. We first infer from Proposition 1 that: (1) If h is large, part substitution is more preferred than LOT buy because it is uneconomical to stock the original parts. (2) if c_s is small, part substitution is less sensitive to h because the product is mainly supplied from substitute parts. However, if c_s is larger, part substitution is more sensitive to h as it uses a larger bridge-buy order.

Since the fixed costs are simple additive terms in the profit functions, the LOT buy quantity and the bridge-buy quantity with part substitution are not functions of the fixed cost if both quantities are non-zero. In other words, for a non-trivial LOT buy and a non-trivial part substitution, we have:

Proposition 2. *The LOT buy has a non-zero order quantity, Q_1^*, and the part substitution has a non-zero bridge-buy quantity, Q_2^*, where (1) $Q_1^* > Q_2^*$; (2) $\Pi_{SUB-BB,0}(Q_2^*) - \Pi_{LOT,0}(Q_1^*) > -\sum_{i=1}^{N}\alpha^{i-1}(p_i + b - c_s)\mu_i$; (3) $\Pi_{LOT,0}(Q_1^*) > \Pi_{SUB-BB,0}(Q_2^*)$.*

The first two parts of the above proposition confirm the flexibility and the value of the substitute part. Namely, mixed substitution makes a bridge buy order which is smaller than the order quantity in the LOT buy, leaving more opportunity to use the more flexible substitute parts and generating more profit with the two parts than using the original part only in the LOT buy. The third part is less intuitive; it states that the profit generated from the original parts in the LOT buy is higher than that generated from the original parts in the mixed substitution solution. Parts (2) and (3) in Proposition 2 are both critical to Proposition 3, which summarizes the choice between LOT buy and part substitution.

Starting from Proposition 2, we can outline the optimal sourcing strategies between LOT buy and part substitution as follows:

Proposition 3.

(1) *Mixed substitution outperforms a LOT buy and a pure substitution only if*

$$\Pi_{SUB-BB,0}(Q_2^*) > k_o \quad \text{and} \quad \Pi_{SUB-BB,0}(Q_2^*) - \Pi_{LOT,0}(Q_1^*) + \sum_{i=1}^{N} \alpha^{i-1}(p_i + b - c_s)\mu_i > k_s.$$

(2) *If the above two conditions are not true, we will not adopt a mixed substitution, and* (a) *if* $\sum_{i=1}^{N}\alpha^{i-1}(p_i+b-c_s)\mu_i > k_s$ *and* $\sum_{i=1}^{N}\alpha^{i-1}(p_i+b-c_s)\mu_i - k_s > \Pi_{LOT,0}(Q_1^*) - k_o$, *then we adopt pure substitution*; (b) *if* $\Pi_{LOT,0}(Q_1^*) > k_o$ *and* $\Pi_{LOT,0}(Q_1^*) - k_o > \sum_{i=1}^{N}\alpha^{i-1}(p_i+b-c_s)\mu_i - k_s$, *then we adopt LOT buy*; (c) *otherwise, if* $\sum_{i=1}^{N}\alpha^{i-1}(p_i+b-c_s)\mu_i < k_s$ *and* $\Pi_{LOT,0}(Q_1^*) < k_o$, *then we do nothing.*

We then denote

$$\delta_1 = \Pi_{SUB-BB,0}(Q_2^*),$$

$$\delta_2 = \Pi_{SUB-BB,0}(Q_2^*) - \Pi_{LOT,0}(Q_1^*) + \sum_{i=1}^{N}\alpha^{i-1}(p_i+b-c_s)\mu_i,$$

$$\delta_3 = \Pi_{LOT,0}(Q_1^*), \quad \delta_4 = \sum_{i=1}^{N}\alpha^{i-1}(p_i+b-c_s)\mu_i$$

From Proposition 2 we have $\delta_1 < \delta_3$ and $\delta_2 < \delta_4$ where $\delta_3 - \delta_1 = \delta_4 - \delta_2$. From Proposition 3, the optimal sourcing strategies partition the two-dimensional $k_s - k_d$ space as in Fig. 1.

In the above figure, each area corresponds to a specific sourcing strategy. The existence of the "do nothing" strategy is due to the fixed costs associated with the sourcing strategies.

Line redesign is a more complicated creation process than LOT buy and part substitution. It is clear that a sufficient condition for optimality of line redesign is that future demand generates revenues larger than the associated fixed cost. We further have:

Proposition 4. *Denoting the optimal timing of line redesign as* n^*, *if* $k_d\alpha^{n^*-1} > k_s$ *while* $c_d \geqslant c_s$, *then part substitution always outperforms line redesign.*

Proposition 4 identifies some cases where line redesign is not economical. That is, if the discounted fixed cost and the variable cost of line redesign are both higher than those of part substitution, then we do not need to consider line redesign. In reality, k_d is usually much larger than k_s such that $k_d\alpha^{n^*-1} > k_s$ holds. Therefore, in the numerical experiments, we will focus on the cases where $c_s > c_d$; otherwise, the effective region for line redesign will be limited.

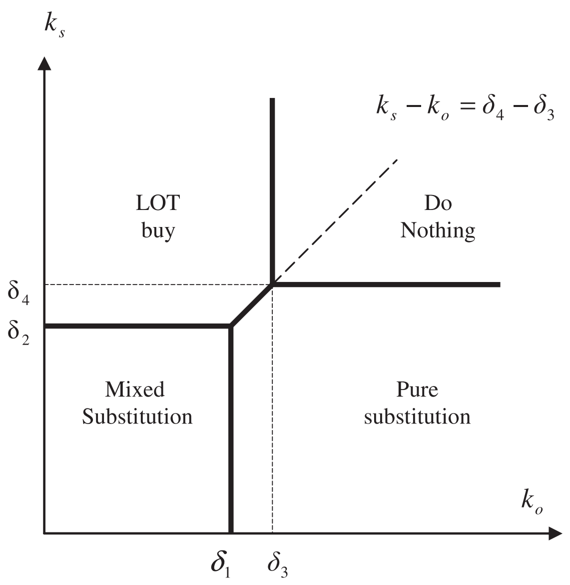

Fig. 1. Boundaries between LOT buy and part substitution.

4. An integrated sourcing approach

In this section, we propose an integrated sourcing strategy which combines and evaluates all the possible sourcing approaches throughout the product's life. This strategy starts with a bridge buy and then takes a possible recourse action when the inventories of original parts are depleted. The time of the possible recourse action is determined by the realized demand; i.e. it is path dependent. The possible actions include supplying the products by a substitute part, designing out the original part and replacing it with a newly designed part, and taking no action. The optimal choice is decided by the expected future profit associated with each option. For example, if the original parts are depleted near the end of the product life, it is likely optimal to take no action as there is fixed cost associated with either part substitution or line redesign. An integrated sourcing solution is more flexible than any static policy, but it is unclear if it must be better than the static ones due to the existence of the fixed costs.

With the same notations introduced in Section 3, we assume the bridge-buy quantity as Q_4, and formulate an integrated sourcing model as follows:

Problem INT

$$Max_{Q_4\geqslant 0}\Pi_{INT}(Q_4) = \Pi_{INT,1}(Q_4) + \sum_{n=1}^{N-1}\Pi_{INT,2}(Q_4|n) \tag{12}$$

$$\begin{aligned}\Pi_{INT,1}(Q_4) = -k_o - c_oQ_4 + E_{X_1\ldots X_N}\Bigg[&\sum_{i=1}^{N}\alpha^{i-1}p_i\min\left(\left(Q_4-\sum_{j=1}^{i-1}X_j\right)^+, X_i\right)\\ &-h\sum_{i=1}^{N-1}\alpha^{i-1}\left(Q_4-\sum_{j=1}^{i}X_j\right)^+ + v\alpha^{N-1}\left(Q_4-\sum_{j=1}^{N}X_j\right)^+\\ &-b\alpha^{N-1}1_{\sum_{j=1}^{N-1}X_j<Q_4}\left(\sum_{j=1}^{N}X_j-Q_4\right)^+\Bigg]\end{aligned} \tag{13}$$

$$\Pi_{INT,2}(Q_4|1\leqslant n\leqslant N-1) = E_{X_1\cdots X_n}1_{\sum_{j=1}^{n-1}X_j<Q_4}1_{\sum_{j=1}^{n}X_j\geqslant Q_4}\left[-b\alpha^{n-1}\left(\sum_{j=1}^{n}X_j-Q_4\right)^+ + \pi_n\right] \tag{14}$$

where

$$\pi_n = \max(\pi_{ns}, \pi_{nd}, \pi_{no})\ (1\leqslant n\leqslant N-1) \tag{15}$$

$$\pi_{ns} = -k_s\alpha^n + \sum_{j=n+1}^{N}\alpha^{j-1}(p_j-c_s)\mu_j \quad (1\leqslant n\leqslant N-1) \tag{16}$$

$$\pi_{nd} = -k_d\alpha^{n+L_2} - b\sum_{j=n+1}^{n+L_2-1}\alpha^{j-1}\mu_j + \sum_{j=n+L_2}^{N}\alpha^{j-1}(p_j-c_d)\mu_j \quad (1\leqslant n\leqslant N-L_2-1) \tag{17}$$

$$\pi_{no} = -b\sum_{j=n+1}^{N}\alpha^{j-1}\mu_j \quad (1\leqslant n\leqslant N-1) \tag{18}$$

The step function, $1_{x\geqslant y} = 1$ if $x \geqslant y$ and $1_{x\geqslant y} = 0$ otherwise. In (14), this step function takes effect in period n where in period n-1 there is still leftover from the bridge buy but in period n the original parts are exactly depleted, which is followed by a possible recourse action if $n < N$. Second, the leadtime for part substitution is again one period; therefore, it could be used in periods $n + 1$ to N if it was initiated in period n. The same logic applies to line redesign. Third, $\Pi_{INT,1}(Q_4)$ is the profit generated from the bridge buy. If the original parts are depleted in period n ($n < N$), then a penalty cost is incurred in period n while initiating the recourse action at the end of period n; this generates an expected future profit π_n. The sum of the penalty and the future profit is denoted as $\Pi_{INT,2}(Q_4|n)$. The recourse actions include the trivial one which takes no action but incurs a penalty cost. The optimal recourse action is to take the most profitable one of π_{ns}, π_{nd}, π_{no}, corresponding to part substitution, line redesign and no action. From (15)–(18), it is only a computational process to find π_n for each n. Incorporating the demand distribution functions in Section 3, we have

Proposition 5. *The profit function in the integrated sourcing approach is*

$$\begin{aligned}\Pi_{INT}(Q_4) = &-k_o + (p_1 + b - c_o)Q_4 \\ &- \sum_{i=1}^{N-1} \alpha^{i-1}(p_i - \alpha p_{i+1} + b(1-\alpha) + h) \\ &\times \int_0^{Q_4} F_{Y_{1i}}(x)\,dx - \alpha^{N-1}(p_N + b - v) \\ &\times \int_0^{Q_1} F_{Y_{1N}}(x)\,dx + (\pi_1 - b\mu_1) \\ &- (\pi_{N-1} + \alpha^{N-1} b\mu_N)F_{Y_{1N-1}}(Q_4) \\ &+ \sum_{i=1}^{N-2}(\pi_{i+1} - \pi_i - \alpha^i b\mu_{i+1})F_{Y_{1i}}(Q_4) \end{aligned} \qquad (19)$$

which is non-concave under many conditions. However, the optimal solution, Q_4^*, still satisfies its first-order condition:

$$\begin{aligned}(p_1 + b - c_o) &- \sum_{i=1}^{N-1} \alpha^{i-1}[p_i - \alpha p_{i+1} + b(1-\alpha) + h]F_{Y_{1i}}(Q_4^*) \\ &- \alpha^{N-1}(p_N + b - v)F_{Y_{1N}}(Q_4^*) \\ &- (\pi_{N-1} + \alpha^{N-1} b\mu_N)f_{Y_{1N-1}}(Q_4^*) \\ &+ \sum_{i=1}^{N-2}(\pi_{i+1} - \pi_i - \alpha^i b\mu_{i+1})f_{Y_{1i}}(Q_4^*) = 0 \end{aligned} \qquad (20)$$

Some direct numerical examples can verify that $\Pi_{INT}(Q_4)$ is non-concave and it often has multiple extreme points when Q_4 is small. Nevertheless, $\Pi_{INT}(Q_4)$ has regular asymptotic conditions when $Q_4 \to 0$ or ∞, which says the optimal solution must be given by the first-order condition and it is finite. Since it is a one-variable function, it is easy to find the optimal solution with current numerical software.

5. Numerical studies

It is not easy to gain further analytical insights from the models derived in Sections 3 and 4. We therefore resort to numerical approaches. This section starts with comparative studies on the three static sourcing approaches and then turns to the integrated approach. We first fix certain problem parameters by setting c_o = \$3.0, v = \$ 1.0, c_d = \$3.5. The fixed costs of the original supplier, substitute-part supplier and line redesign are all additive in the profit functions; therefore, we choose $k_o = 0$, and k_s = \$50,000 throughout this section because k_o is often relatively small and we can always normalize the three fixed costs to make it zero without affecting the final results. These data closely reflect reality in the military and avionics sectors (Livingston, 2000). Finally, consistent with Proposition 4, we focus on the situations in which $c_s > c_d$ and $k_d > k_s$. We study a 12-period model in which the demands are uncorrelated and normally distributed

$$\{\mu_1, \ldots, \mu_{12}\} = \{10,000, 10,000, 10,000, 30,000, 30,000, 30,000, 30,000, 30,000, 30,000, 5000, 5000, 5000\}$$

$$\{\sigma_1, \ldots, \sigma_{12}\} = \{4000, 4000, 4000, 12,000, 12,000, 12,000, 12,000, 12,000, 12,000, 2000, 2000, 2000\}.$$

This demand data says the product is still at an early stage of its lifecycle. We set the product prices as $p_1 \cdots p_N$ = \$10.

5.1. Static sourcing strategies

Starting from Proposition 1, in this section we will tease out the joint role that the problem parameters play. With the primary understanding between LOT buy and part substitution in Section 3.5, we mainly look into line redesign versus LOT buy and part substitution. To this end, we will choose combinations of h = {\$0.1,\$0.4} and c_s = {\$4,\$7} throughout this section. The penalty cost, b, is set as \$5 and fixed in this subsection, knowing this factor is most sensitive for LOT buy. The discount factor is first chosen as α = 0.95. We also set $L_2 = 2$, which means the period to adopt the newly designed part, n, is no earlier than period 2. An optimal solution to line redesign may delay n to a period later than period 2.

With all the given data as a base case, in Fig. 2 we vary the fixed cost for line redesign and then present its corresponding profits in relation to the other two sourcing options.

When the fixed cost, k_d, is large, it approaches LOT buy as expected. We further have:

(1) If k_d is still small, the profit of a nontrivial line redesign is slightly sensitive to h, implying that fewer original parts are used in line redesign. However, if k_d is large but h is small, line redesign quickly reduces to LOT buy, whereas this happens much slower if h is large. This indicates that line redesign is valuable if it is uneconomical to stockpile the original parts.

(2) Between line redesign and part substitution, if the variable costs, c_s and c_d, are close then line redesign is dominated by part substitution, due to line redesign's higher fixed cost.

(3) If c_s is large, line redesign outperforms part substitution for a greater range of k_d if h increases. The reason is that part substitution has to carry a significant bridge-buy quantity if c_s is large, while line redesign requires fewer units of the original parts.

Understanding the joint effects of the cost factors on the three static approaches, we move onto other problem determinants. As noted earlier, financial discount factor is important for products over a long period of time while the profit in each sourcing option will decrease as the discount factor decreases. We want to know the relative impact of this factor on the three strategies. We therefore change α from 0.95 to 0.8 while preserving all other parameters as above, with the results shown in Fig. 3.

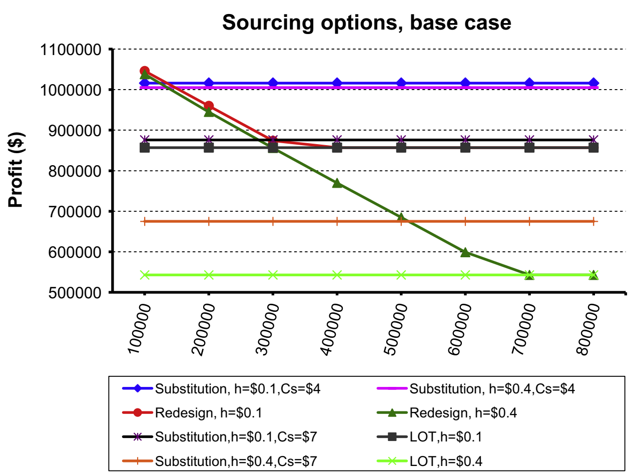

Fig. 2. Base case.

The major finding from Fig. 3 is that a lower discount factor significantly reduces the sensitivity to h for part substitution and line redesign. In fact, both part substitution and line redesign change little when h is varied from of \$0.1 to \$0.4, whether c_s is high or low. However, LOT buy is still quite sensitive to h. The interpretation is that a lower discount factor will drive line redesign and part substitution to rely more heavily on the newly designed parts and the substitute parts respectively, since the costs of these parts will be discounted more sharply. The LOT-buy part cost is calculated without discount, so it is more sensitive to h. The above observations imply that redesign is more likely to be adopted if the discount factor decreases.

We then try to understand the role of leadtime in the problem by increasing L_2 for line redesign from 2 periods to 7 periods while keeping other parameters as in Fig. 2. The results are presented in Fig. 4. For a relatively long leadtime, L_2, we find that line redesign is now quite sensitive to the inventory holding cost of the original parts. The reason is that the longer leadtime necessitates a larger bridge-buy order. In addition, a longer redesign leadtime automatically restricts us to a later time to use the newly designed parts, which requires a smaller fixed cost for line redesign to be economically viable, particularly when h is high.

5.2. Static versus integrated models

While Problem INT looks more complicated than the static models, LOT buy still provides a lower bound for this sourcing approach. We preserve the fixed costs, k_0, k_s, the demand profile and

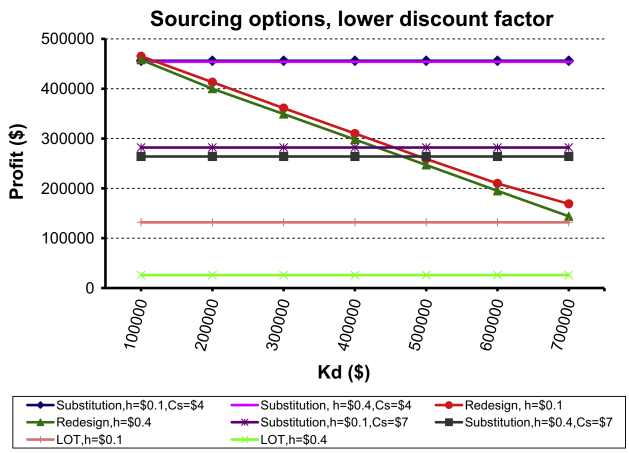

Fig. 3. Lower discount factor.

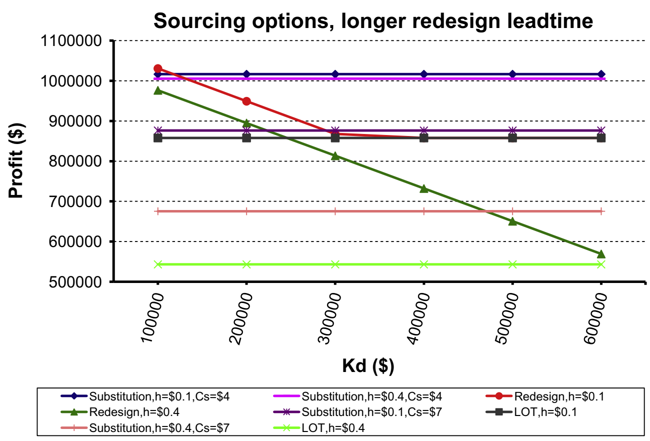

Fig. 4. Longer redesign leadtime.

product prices in this section while setting k_d = \$300,000, c_o = \$7, c_s = \$9, c_d = \$5, h = \$0.1. Noting the basic properties of b and L_2 from Proposition 1, we also set b = \$0 and L_2 = 2. Through our numerical analysis, we find the discount factor is the most critical factor that determines when the integrated sourcing approach is the optimal mitigation strategy.

In Fig. 5, we demonstrate this statement by choosing α = 0.95, 0.85, 0.75. From Fig. 5, an integrated sourcing approach is in general more profitable than the three static solutions when α becomes small. It could be as much as 82% more profitable when α = 0.75 and c_d = \$7. On the other hand, if α is large, an integrated sourcing approach might be worse than part substitution or line redesign. The reason for this occurrence is that the fixed cost of part substitution or line redesign cannot offset the revenue from the recourse action on average.

5.3. Summary from the numerical results

We have conducted additional numerical studies by varying the problem parameters along all possible dimensions. In summary, the integrated sourcing approach can outperform the three static solutions under many circumstances but it does not dominate them. This also explains why the static solutions prevail in practice, not only because they are simple to manage but also because they are more profitable in many cases.

Figs. 1–5 have well depicted the appropriate sourcing approach when one part is subject to obsolescence. As stated in Porter (1998), line redesign is usually not favorable because of its technical requirements and its cost disadvantage. In the current one-part models, we find line design is an optimal solution only if the associated fixed cost is sufficiently small. The advantage of line redesign lies in its ability to resolve multi-part obsolescence when a high fixed cost can be shared by a number of parts. In terms of LOT buy and part substitution, the latter is economically more favorable unless the fixed cost to use the substitute parts is comparably too high. These general observations support the solution adoption frequencies observed in industry. However, we have also shown that every solution could be an optimal choice in a specific case. We also have found that the financial discount factor plays a critical role in determining the right supplier. If the financial discount factor is lower, line redesign or substitution is more likely adopted in either a static or integrated setting.

6. Conclusions and future research

As parts made for consumer-product markets continue to be adopted into the industrial sector, the importance of properly managing part obsolescence will continue to increase. We have studied the three most frequently adopted static sourcing strategies to cope with part obsolescence by assuming there is only one part that is subject to obsolescence and there is only one obsolescence event. The analytical and numerical results explain why different obsolescence solutions have been adopted in practice. We further propose an integrated sourcing approach starting with a bridge buy of the original parts with a possible recourse action of part substitution or line redesign. Such a flexible approach might be a better solution than the three static approaches under certain conditions. As a result, running the four models in this paper can tell us which one is the best choice. The models in this paper can serve as decision tools in determining the optimal sourcing strategy to combat the part obsolescence problem.

We can extend the models in Section 3 under more general conditions. First, it may not always be the case that the selling price in each period exceeds the part costs. For instance, it is possible that the substitute part cost or the cost of the new part from line

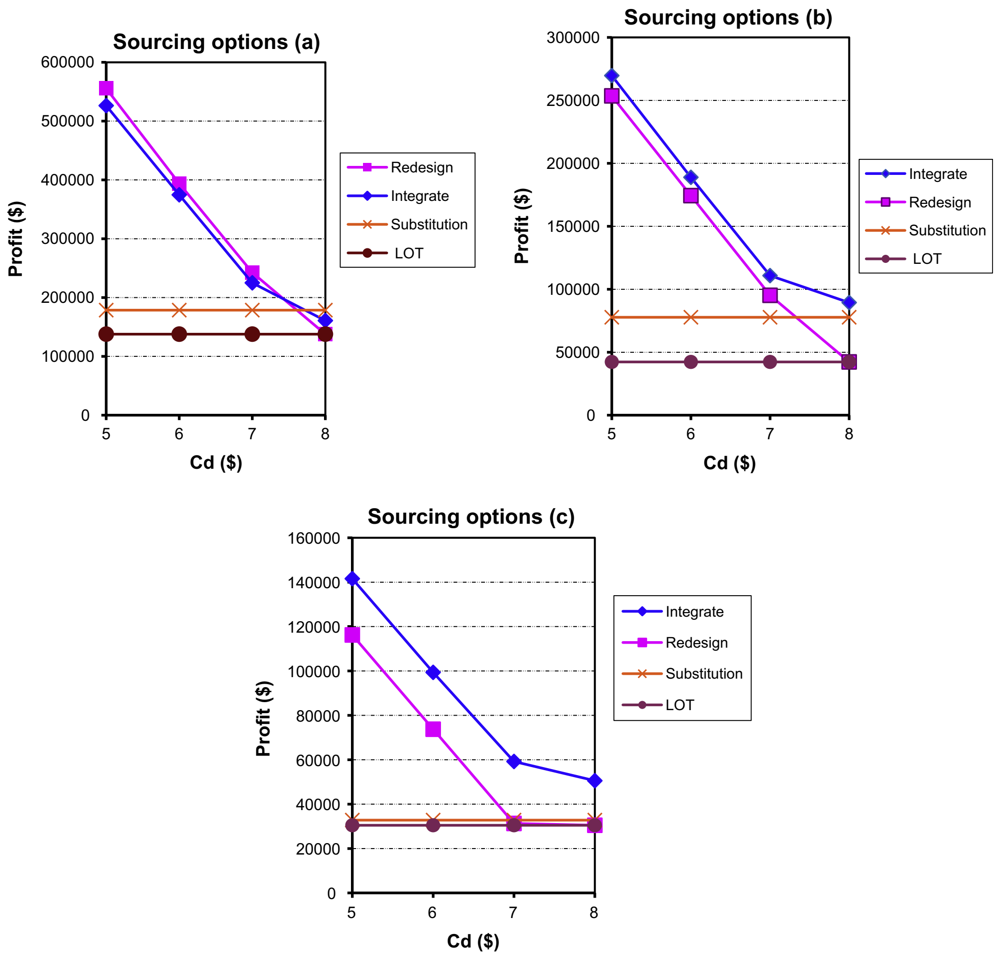

Fig. 5. Integrated sourcing approach against static sourcing approaches (a) $\alpha = 0.95$; (b) $\alpha = 0.85$; (c) $\alpha = 0.75$.

redesign could exceed the product's price in some periods. These situations can be modeled in a similar way as in the static models. Second, it is possible that the leadtime of the substitute part is longer than one period, when some demand might not be fulfilled by the bridge buy before the substitute parts arrive. This implies that Propositions 2 and 3 need modification.

Finally, there are several directions for future research:

(1) In practice, the product prices throughout the planning horizon might not be certain. The end of the product lifecycle might also be uncertain. For both cases, the resulting problem will be more complex but this work can serve as a starting point.
(2) In another way, the OEM may consider the "planned discontinuity" of the product itself if the rest of the product lifecycle is not so long but a part obsolescence solution other than LOT buy is not economical. Such a solution has not been presented in the literature nor documented in practice. Research in this direction will add further venue.
(3) All the sourcing approaches in this paper are proposed at the time when part obsolescence occurs; therefore, they are reactive mitigation approaches. This is natural when only one obsolescence event happens. If there are multiple obsolescence events in sequence, we need to plan part obsolescence in a rolling horizon framework. This may happen to the case of part substitution where the replacement part might become obsolete in the future itself.
(4) Although line redesign appears less cost effective than other solutions for one-part obsolescence, it is interesting to study such a solution against other solutions, like LOT buys in sequence. There are some works toward this goal; however, a systematic modeling approach is still absent.
(5) The data set chosen in the numerical studies in Section 5 is adapted from the avionics and military sectors. It is interesting to adapt data from those industrial sectors.

Acknowledgements

We would like to thank the editor and three anonymous referees for many constructive comments. The research of the first author is supported by National Science Foundation of China (Grant No. 71071091).

Appendix A. Proof of Lemma 1

From (1) in the text, we have

$$\frac{\partial^2 \Pi_{LOT}(Q_1)}{\partial^2 Q_1} = -\sum_{i=1}^{N-1} \alpha^{i-1}[p_i + b(1-\alpha) + h - \alpha p_{i+1}] f_{Y_{1,i}}(Q_1) - \alpha^{N-1}(p_N + b - v) f_{Y_{1,N}}(Q_1) < 0 \quad \text{(A1)}$$

Hence $\Pi_{LOT}(Q_1)$ is concave and the first-order condition, stated as (4) in the text, provides the optimal solution.

Appendix B. Proof of Lemma 2

(1) The proof is in parallel with Lemma 1, so the detail is omitted.
(2) From (4) and (8) in the text, we can find these two asymptotic behaviors.

Appendix C. Proof of Lemma 3

(1) The proof is similar to Lemma 1 though it is much more involved. We omit the detail.
(2) For a fixed n, the local optimal order quantity is provided by (11). Since n is finite, we can enumerate on n and find the global optimal order quantity and then the optimal timing.

Appendix D. Proof of Proposition 1

(1) From (4), it is obvious that if h increases, Q_1^* will decrease. Therefore, $\partial Q_1^*/\partial h < 0$.
Taking derivative of (4) with respect to h while inserting $Q_1 = Q_1^*$, we yield

$$\frac{\partial \Pi_{LOT}(Q_1^*)}{\partial h} = \frac{\partial Q_1^*}{\partial h}\frac{\partial \Pi_{LOT}(Q_1)}{\partial Q_1}\Big|_{Q_1=Q_1^*} + \frac{\partial \Pi_{LOT}(Q_1)}{\partial h}\Big|_{Q_1=Q_1^*} = -\sum_{i=1}^{N-1}\alpha^{i-1}\int_0^{Q_1^*}F_{Y_{1,i}}(x)\,dx < 0$$

Therefore, we have proved the first half of the first part of the proposition.
Regarding the effect of the penalty cost, we first take a partial derivative of the left side of (4) in the text with respect to b, which yields

$$\begin{aligned}1 - \sum_{i=1}^{N-1}\alpha^{i-1}(1-\alpha)F_{Y_{1,i}}(Q_1^*) - \alpha^{N-1}F_{Y_{1,N}}(Q_1^*) &= \sum_{i=1}^{N-1}\alpha^{i-1}[p_i + b(1-\alpha) + h - \alpha p_{i+1}]f_{Y_{1,i}}(Q_1^*)\frac{\partial Q_1^*}{\partial b}\\ &\quad + \alpha^{N-1}(p_N + b - v)f_{Y_{1,N}}(Q_1^*)\frac{\partial Q_1^*}{\partial b}\end{aligned} \tag{A2}$$

But we have $1 - \sum_{i=1}^{N-1}\alpha^{i-1}(1-\alpha)F_{Y_{1,i}}(Q_1^*) - \alpha^{N-1}F_{Y_{1,N}}(Q_1^*) > 1 - \sum_{i=1}^{N-1}\alpha^{i-1}(1-\alpha) - \alpha^{N-1} = 0$ for the left side of (A2); therefore, from the right side of (A2) together with Assumption 1 we have $\partial Q_1^*/\partial b > 0$.
Then, from (1) we have

$$\begin{aligned}\frac{\partial \Pi_{LOT}(Q_1^*)}{\partial b} &= \frac{\partial Q_1^*}{\partial b}\frac{\partial \Pi_{LOT}(Q_1)}{\partial Q_1}\Big|_{Q_1=Q_1^*} + \frac{\partial \Pi_{LOT}(Q_1)}{\partial b}\Big|_{Q_1=Q_1^*}\\ &= -\sum_{i=1}^{N}\alpha^{i-1}\mu_i + Q_1^* - \sum_{i=1}^{N-1}\alpha^{i-1}(1-\alpha)\int_0^{Q_1^*}F_{Y_{1,i}}(x)dx\\ &\quad - \alpha^{N-1}\int_0^{Q_1^*}F_{Y_{1,N}}(x)\,dx < -\sum_{i=1}^{N}\alpha^{i-1}\mu_i + Q_1^*\\ &\quad - \sum_{i=1}^{N-1}\alpha^{i-1}(1-\alpha)\left(Q_1^* - \sum_{j=1}^{i}\mu_j\right) - \alpha^{N-1}\left(Q_1^* - \sum_{j=1}^{N}\mu_j\right) = 0\end{aligned}$$

We also have

$$\frac{\partial \Pi_{LOT,0}(Q_1^*)}{\partial b} = Q_1^* - \sum_{i=1}^{N-1}\alpha^{i-1}(1-\alpha)\int_0^{Q_1^*}F_{Y_{1,i}}(x)dx - \alpha^{N-1}\int_0^{Q_1^*}F_{Y_{1,N}}(x)\,dx > 0$$

Therefore, the second half of the first part of this proposition is proved too.

(2) The detail follows the above, so it is omitted.
(3) The detail follows the above, so it is omitted.

Appendix E. Proof of Proposition 2

(1) We can rewrite (4) and (8) as

$$p_1 + b - c_o = \sum_{i=1}^{N-1}\alpha^{i-1}[p_i + b(1-\alpha) + h - \alpha p_{i+1}]F_{Y_{1,i}}(Q_1^*) + \alpha^{N-1}(p_N + b - v)F_{Y_{1,N}}(Q_1^*) \tag{A3}$$

$$c_s - c_o = [h + (1-\alpha)c_s]\sum_{i=1}^{N-1}\alpha^{i-1}F_{Y_{1,i}}(Q_2^*) + \alpha^{N-1}(c_s - v)F_{Y_{1,N}}(Q_2^*) \tag{A4}$$

We first prove $Q_1^* > Q_2^*$ by contradiction. If this is not true (i.e. $Q_1^* < Q_2^*$), then from (A3) we have

$$p_1 + b - c_o < \sum_{i=1}^{N-1}\alpha^{i-1}[p_i + b(1-\alpha) + h - \alpha p_{i+1}]F_{Y_{1,i}}(Q_{21}^*) + \alpha^{N-1}(p_N + b - v)F_{Y_{1,N}}(Q_2^*)$$

Subtracting the above by (A4), we obtain

$$\begin{aligned}p_1 + b - c_s &< \sum_{i=1}^{N-1}\alpha^{i-1}[p_i + (b - c_s)(1-\alpha) - \alpha p_{i+1}]F_{Y_{1,i}}(Q_2^*)\\ &+ \alpha^{N-1}(p_N + b - c_s)F_{Y_{1,N}}(Q_2^*) < \sum_{i=1}^{N-1}\alpha^{i-1}[p_i + (b - c_s)(1-\alpha) - \alpha p_{i+1}]\\ &+ \alpha^{N-1}(p_N + b - c_s) = p_1 + b - c_s\end{aligned}$$

which is contradictory, and therefore $Q_1^* > Q_2^*$ must be true.

(2) We further have

$$\begin{aligned}\Pi_{LOT,0}(Q_1^*) &= (p_1 + b - c_o)Q_1^* - \sum_{i=1}^{N-1}\alpha^{i-1}[p_i + h + b(1-\alpha)\\ &-\alpha p_{i+1}]\int_0^{Q_1^*}F_{Y_{1,i}}(x)dx - \alpha^{N-1}(p_N + b - v)\int_0^{Q_1^*}F_{Y_{1,N}}(x)dx\,\Pi_{SUB-BB,0}(Q_2^*)\\ &= (c_s - c_o)Q_2^* - [h + (1-\alpha)c_s]\sum_{i=1}^{N-1}\alpha^{i-1}\int_0^{Q_2^*}F_{Y_{1,i}}(x)dx\\ &-\alpha^{N-1}(c_s - v)\int_0^{Q_2^*}F_{Y_{1,N}}(x)\,dx\end{aligned}$$

Therefore,

$$\begin{aligned}\Pi_{SUB-BB,0}(Q_2^*) - \Pi_{LOT,0}(Q_1^*) &= (c_s - c_o)Q_2^* - (p_1 + b - c_o)Q_1^*\\ &+ \sum_{i=1}^{N-1}\alpha^{i-1}[p_i + h + b(1-\alpha) - \alpha p_{i+1}]\int_0^{Q_1^*}F_{Y_{1,i}}(x)\,dx\\ &+ \alpha^{N-1}(p_N + b - v)\int_0^{Q_1^*}F_{Y_{1,N}}(x)\,dx\\ &- [h + (1-\alpha)c_s]\sum_{i=1}^{N-1}\alpha^{i-1}\int_0^{Q_2^*}F_{Y_{1,i}}(x)dx - \alpha^{N-1}(c_s - v)\int_0^{Q_2^*}F_{Y_{1,N}}(x)\,dx\end{aligned}$$

With some manipulations, we can write $\Pi_{SUB-BB,0}(Q_2^*) - \Pi_{LOT,0}(Q_1^*) = A + B$, where

$$\begin{aligned}A &= \sum_{i=1}^{N-1}\alpha^{i-1}[p_i - \alpha p_{i+1} + (1-\alpha)(b - c_s)]\int_{Q_1^*}^{\infty}xf_{Y_{1,i}}(x)\,dx\\ &+ \alpha^{N-1}(p_N + b - c_s)\int_{Q_1^*}^{\infty}xf_{Y_{1,N}}(x)\,dx - \sum_{i=1}^{N-1}\alpha^{i-1}[h + (1-\alpha)c_s]\\ &\int_{Q_2^*}^{Q_1^*}xf_{Y_{1,i}}(x)dx - \alpha^{N-1}(c_s - v)\int_{Q_2^*}^{Q_1^*}xf_{Y_{1,N}}(x)\,dx\end{aligned}$$

$$\begin{aligned}B &= -\sum_{i=1}^{N-1}\alpha^{i-1}[p_i - \alpha p_{i+1} + h + b(1-\alpha)]\sum_{j=1}^{i}\mu_j - \alpha^{N-1}(p_N + b - c_s)\\ &\times\sum_{j=1}^{N}\mu_j + \sum_{i=1}^{N-1}\alpha^{i-1}[h + (1-\alpha)c_s]\sum_{j=1}^{i}\mu_j + \alpha^{N-1}(c_s - v)\sum_{j=1}^{N}\mu_j\end{aligned}$$

We first prove $A \geqslant 0$. Since $Q_1^* > Q_2^*$, we have

$$\begin{aligned}A \geqslant &\sum_{i=1}^{N-1}\alpha^{i-1}[p_i - \alpha p_{i+1} + (1-\alpha)(b-c_s)]Q_1^*\left[1 - F_{Y_{1,i}}(Q_1^*)\right] \\ &+ \alpha^{N-1}(p_N + b - c_s)Q_1^*\left[1 - F_{Y_{1,i}}(Q_1^*)\right] \\ &- \sum_{i=1}^{N}\alpha^{i-1}[h + (1-\alpha)c_s]Q_1^*\left[F_{Y_{1,i}}(Q_1^*) - F_{Y_{1,i}}(Q_2^*)\right] \\ &- \alpha^{N-1}(c_s - v)Q_1^*\left[F_{Y_{1,N}}(Q_1^*) - F_{Y_{1,N}}(Q_2^*)\right]\end{aligned}$$

Iteratively using (A3) and (A4) to simplify the right-hand side of the above identity, we find it is indeed equal to zero, and therefore $A \geqslant 0$.

Next we consider B. Taking advantage of

$$\sum_{i=1}^{N-1}\alpha^{i-1}(p_i - \alpha p_{i+1})\sum_{j=1}^{i}\mu_j = \sum_{i=1}^{N-1}\alpha^{i-1}p_i\mu_i - \alpha^{N-1}p_N\sum_{i=1}^{N-1}\mu_i$$

$$\sum_{i=1}^{N-1}\alpha^{i-1}\sum_{j=1}^{i}\mu_j = \frac{1}{1-\alpha}\sum_{i=1}^{N-2}\alpha^{i-1}\mu_i - \frac{\alpha^{N-1}}{1-\alpha}\sum_{i=1}^{N-2}\mu_i + \alpha^{N-2}\mu_{N-1}$$

we have

$$\begin{aligned}B = &-\sum_{i=1}^{N-1}\alpha^{i-1}p_i\mu_i + \alpha^{N-1}p_N\sum_{i=1}^{N-1}\mu_i - (b-c_s) \\ &\times\left[\sum_{i=1}^{N-2}\alpha^{i-1}\mu_i - \alpha^{N-1}\sum_{i=1}^{N-2}\mu_i + (1-\alpha)\alpha^{N-2}\mu_{N-1}\right] \\ &- \alpha^{N-1}(p_N + b - c_s)\sum_{i=1}^{N}\mu_i = -\sum_{i=1}^{N}\alpha^{i-1}(p_i + b - c_s)\mu_i\end{aligned}$$

Finally, we have $\Pi_{SUB-BB,0}(Q_2^*) - \Pi_{LOT,0}(Q_1^*) > -\sum_{i=1}^{N}\alpha^{i-1}(p_i + b - c_s)\mu_i$.

(3) Since $\partial\Pi_{LOT,0}(Q_1^*)/\partial b > 0$ from Proposition 1, if we can prove $\Pi_{LOT,0}(Q_1^*) > \Pi_{SUB-BB,0}(Q_2^*)$ when $b = 0$, then the statement is true for any non-zero b.

Writing Q_{10}^* for Q_1^* when $b = 0$, we can rewrite:

$$\begin{aligned}\Pi_{LOT,0}(Q_{10}^*) = &(p_1 - c_o)Q_{10}^* - \sum_{i=1}^{N-1}\alpha^{i-1}[p_i + h - \alpha p_{i+1}]\int_0^{Q_{10}^*}F_{Y_{1,i}}(x)\,dx \\ &- \alpha^{N-1}(p_N + b - v)\int_0^{Q_{10}^*}F_{Y_{1,N}}(x)\,dx\,\Pi_{SUB-BB,0}(Q_2^*) \\ = &(c_s - c_o)Q_2^* - [h + (1-\alpha)c_s]\sum_{i=1}^{N-1}\alpha^{i-1}\int_0^{Q_2^*}F_{Y_{1,i}}(x)\,dx \\ &- \alpha^{N-1}(c_s - v)\int_0^{Q_2^*}F_{Y_{1,N}}(x)\,dx\end{aligned}$$

Since Q_{10}^* is the maximal value for $\Pi_{LOT,0}(Q_{10}^*)$ while $Q_{10}^* > Q_2^*$ from the first part of this proposition, we must have $\Pi_{LOT,0}(Q_{10}^*) > \Pi_{LOT,0}(Q_2^*)$. However,

$$\begin{aligned}\Pi_{LOT,0}(Q_2^*) - \Pi_{SUB-BB,0}(Q_2^*) = &(p_1 - c_s)Q_2^* \\ &- \sum_{i=1}^{N-1}\alpha^{i-1}[p_i - \alpha p_{i+1} - (1-\alpha)c_s]\int_0^{Q_2^*}F_{Y_{1,i}}(x)\,dx \\ &- \alpha^{N-1}(p_N - c_s)\int_0^{Q_2^*}F_{Y_{1,N}}(x)\,dx \\ > &Q_2^*\Big\{(p_1 - c_s) - \sum_{i=1}^{N-1}\alpha^{i-1}[p_i - \alpha p_{i+1} \\ &-(1-\alpha)c_s] - \alpha^{N-1}(p_N - c_s)\Big\} = 0\end{aligned}$$

From the previous analysis, for any positive b, we have $\Pi_{LOT,0}(Q_1^*) > \Pi_{SUB-BB,0}(Q_2^*)$.

Appendix F. Proof of Proposition 3

We first repeat the results in Sections 3.2 and 3.3,

(1) If the LOT buy quantity is Q_1^*, its profit is

$$\Pi_{LOT}(Q_1^*) = -k_o - b\sum_{i=1}^{N}\alpha^{i-1}\mu_i + \Pi_{LOT,0}(Q_1^*);$$

(2) The profit generated from a pure substitution is $-k_s + \sum_{i=1}^{N}\alpha^{i-1}(p_i - c_s)\mu_i$;

(3) If the bridge-buy quantity is Q_2^* in a mixed substitution, its profit is

$$\Pi_{SUB-BB}(Q_2^*) = -k_o - k_s + \sum_{i=1}^{N}\alpha^{i-1}(p_i - c_s)\mu_i + \Pi_{SUB-BB,0}(Q_2^*);$$

(4) If taking no action, it incurs a penalty cost of $-b\sum_{i=1}^{N}\alpha^{i-1}\mu_i$. Utilizing the above results with Proposition 2, we can arrive at the results in Proposition 3 by direct algebraic comparisons.

Appendix G. Proof of Proposition 4

If line redesign is finished in period n^*, then the difference between part substitution and line redesign lies in three aspects: (a) part substitution starts in period one, while line redesign starts in period n^*; (b) the fixed cost for part substitution is k_s, while the discounted fixed cost for line redesign is $k_d\alpha^{n^*-1}$; (c) the variable costs of the two parts are c_s, c_d, respectively.

Excluding the fixed costs associated with part substitution and line redesign, we compare the corresponding profits of these two options. We first release the restriction on the leadtime for line redesign such that the new parts can be used from the first period. The profit for such a relaxed problem sets an upper bound for the original problem of line redesign. However, if $c_d > c_s$, then the relaxed line redesign problem must be less profitable than part substitution since the two problems differ only in the variable cost. The composition of these two results declares that even without considering the fixed costs line redesign is less profitable than part substitution if $c_d > c_s$. Therefore, if $k_d\alpha^{n^*-1} > k_s$, then part substitution is more profitable than line redesign.

Appendix H. Proof of Proposition 5

To derive the profit function in Proposition 5, we first have

$$\begin{aligned}&E_{X_1\cdots X_n}1_{\sum_{j=1}^{n-1}X_j<Q_4}1_{\sum_{j=1}^{n}X_j>Q_4}\left(\sum_{j=1}^{n}X_j - Q_4\right)^+ \\ &= \int_{y=Q_4-x}^{\infty}\int_{x=0}^{Q_4}f_{Y_{1n-1}}(x)f_{Y_n}(y)(x+y-Q_4)\,dx\,dy \\ &= \mu_nF_{Y_{1n-1}}(Q_4) + \int_{x=0}^{Q_4}dxF_{Y_{1n}}(x) - \int_{x=0}^{Q_4}dxF_{Y_{1n-1}}(x)\end{aligned}$$

$$\begin{aligned}E_{X_1\cdots X_n}1_{\sum_{j=1}^{n-1}X_j<Q_4}1_{\sum_{j=1}^{n}X_j>Q_4} &= \int_{y=Q_4-x}^{\infty}\int_{x=0}^{Q_4}f_{Y_{1n-1}}(x)f_{Y_n}(y)\,dx\,dy \\ &= F_{Y_{1n-1}}(Q_4) - F_{Y_{1n}}(Q_4)\end{aligned}$$

Inserting the above two identities in Problem INT and combining all the terms, we can drive the result. The calculation is rather lengthy but straightforward, so the detail is omitted here.

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