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## Coordinating a channel with asymmetric cost information and the manufacturer's optimality

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### ABSTRACT

In a manufacturer–retailer system with private retail cost information, we find that a set of incentive-compatible contracts consisting of wholesale and buyback prices can coordinate the channel for any retail cost. We then design two wholesale–buyback contracts by imposing a cutoff point on the retail cost. The first contract maximizes the manufacturer's expected profit while ensuring the channel is coordinated. The second contract assumes the same contractual structure without considering the effect on the channel. Both contracts are exactly solved. We find from numerical study that the manufacturer in the first contract can perform closely to the second one in many cases, and cases exist where both the manufacturer and the channel can do better in the first contract versus the second one.

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### 1. Introduction

Original equipment manufacturers (OEMs) are continuing to outsource portions of their supply chains. Instead of a vertically integrated entity controlling all aspects of the product as it evolves from a concept into a commercially available product, there are now multiple independent companies involved in the process with the OEM acting as the channel master coordinating the entire process. When a supply chain involves independent companies, there is inherent difficulty in coordinating the channel to optimize the system's profitability and determining how to share the channel's profits across the companies.

In order to align incentives between different parties in the supply chain, a wide diversity of contracting strategies have been devised and implemented in industry. This has in turn generated a significant stream of academic research in supply contracts; see the review articles by Tsay et al. (1999) and Cachon (2003). While most research has focused on supply contracts for channels where each party in the supply chain has complete knowledge regarding all the parameters across the channel, there is more and more research designing contracts for supply chains where there is private information held by one party.

Two primary areas where information asymmetries can occur relate to cost and demand information. Works including Lewis and Sappington (1988), Cachon and Lariviere (2001), Ozer and Wei (2006), and Burnetas et al. (2007) consider asymmetric demand

information. Works considering asymmetric cost information include Baron and Myerson (1982), Corbett and deGroot (2000), Ha (2001), and Corbett et al. (2004). In order to mitigate the presence of asymmetric information, these papers have designed a set of incentive-compatible contracts such that the party with private information is induced to select the contract which truthfully reveals the party's private information. This is the mechanism design based on the fundamental revelation principle (Fudenberg and Tirole 2000). Beyond the research on asymmetric cost or demand reviewed above, Chakravarty and Zhang (2007) also study asymmetric capacity information based on mechanism design.

Regarding the asymmetric cost information at the downstream stage, Corbett et al. (2004) compare different manufacturer–retailer contracts in the case of deterministic but price-dependent demand. Ha (2001) considers an environment where the market demand is the sum of price-dependent deterministic and stochastic components. When the manufacturer has complete information regarding the retailer's cost, coordinating the channel is possible through some well-known mechanisms. When the retail cost information is private, an incentive-compatible, nonlinear-pricing contract with a cutoff level policy is proposed. However, it is no longer possible to achieve the channel-optimal solution. A cutoff policy in this case dictates a threshold of the retail cost, above which the manufacturer or the retailer will receive less than the reservation profit and therefore an effective contract will not be issued. Such a cutoff policy is shown optimal due to the fact that both the manufacturer's and retailer's profits are decreasing functions of the retail cost.

Our research is motivated by the practice at a telecommunication company. This company is an OEM in the computer peripherals industry that sells its products through several large distributors.

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The OEM had outsourced the vast majority of its manufacturing operations. The OEM realized that from the distributor's perspective, the OEM's wholesale price was just one aspect of the cost incurred by the distributor. In addition to the wholesale price, the distributor was surely incurring a distribution cost. While the OEM did not know the distributor's exact structure of this cost, the OEM could develop reasonable estimates for the distributor's costs. The OEM employed a wholesale-buyback contract with the distributor. There are two conflicting objectives facing the OEM in its role as the channel master in the supply chain. First, the OEM held the belief that "the most efficient supply chain wins" so the OEM was genuinely interested in understanding how best to coordinate the supply chain. Furthermore, since the OEM was the channel master, "operating in its own best interest" meant that the OEM had the ability to structure the contract terms that would be to its best advantage. This problem setting motivated us to compare two wholesale-buyback contracts under asymmetric cost information in a manufacturer-retailer (distributor) setting. In the first contract, the manufacturer tries to extract the maximum profit under the condition that the channel is coordinated. In the second contract, the manufacturer tries to extract the highest profit without considering the channel performance. We hypothesize that coordinating the channel may not necessarily financially harm the OEM, while acting selfishly may not be the best choice either. Our contributions to the literature will be threefold: first, in contrast to the existing literature, we demonstrate that channel coordination under asymmetric cost information is possible via a specific wholesale-buyback contract; second, we prove that a wholesale-buyback contract is equivalent to a nonlinear-pricing contract under asymmetric cost information; third, we demonstrate that the channel-coordinated contract can benefit the channel yet harm or benefit the manufacturer depending on the circumstances.

Coordinating channels has long been of research interest. Pasternack (1985) is the first to study this problem with stochastic demand. He shows that a buyback policy where a manufacturer offers a retailer partial credit for all unsold goods can optimize the channel's profit. Such a buyback policy allows the decentralized two-stage supply chain to achieve the profit produced by operating the supply chain in a centralized fashion. For a detailed exposition of the many extensions to the Pasternack framework, see Pasternack (2008). For example, Donohue (2000) extends the one-period return policy into a two-period problem setting. Granot and Yin (2005) extends the problem into the case that the demand is price-dependent. Very recently, Xiao et al. (2010) coordinate a channel where the retailer can return the unsold products while the final consumers can also return the goods under their valuation after receiving. Our research studies channel coordination via buyback contract under asymmetric retailer cost information.

The remainder of the paper is structured as follows. Section 2 builds the problem preliminaries by incorporating retail cost into Pasternack's buyback framework in a manufacturer-retailer system, assuming this retail cost is known to each party. In Section 3 we study the situation that the manufacturer only has an estimate of the retailer's cost, i.e., an asymmetric cost information case. We first find that the incentive-compatibility condition from revelation principle with the coordinating condition between the wholesale and buyback prices derived in Pasternack (1985) can warrant channel optimality in the asymmetric information case for any realized retail cost. We then propose a contract to maximize the manufacturer's profit based on this type of coordinated channel. We propose another contract that preserves the contract setting but optimizes the manufacturer's profit by allowing the manufacturer to independently set the wholesale and buyback prices. In Section 4, we conduct numerical experiments to compare the

performance of the manufacturer, the retailer and the channel in the two contracts presented in Section 3, which partially verifies our hypothesis. We conclude and discuss the implication of the paper in Section 5.

## 2. Baseline results in the full information case

To introduce notation and establish the first-best results for this channel, we first present the full information case where the manufacturer has complete knowledge of the retail cost. The resulting buyback contract extends Pasternack (1985) to the case of known retail cost at the retailer. We introduce the notations first. We denote  $s$  as unit manufacturing cost,  $w$  as unit wholesale price,  $p$  as unit selling price,  $c$  as unit incremental retail cost, known or unknown to the manufacturer,  $r$  as unit buyback price. Finally we define  $f(x)$  as the probability density function of the demand and  $F(x)$  as the cumulative distribution function.

As usual, we assume  $s < w < p$  and  $w + c < p$  in order not to incur negative profit for any party. We also assume that the retailer must incur the retail cost  $c$  for each unit that the retailer receives. In order to simplify notation, without loss of generality we have assumed the goodwill cost and the salvage cost are both zero.

When the retail cost is known to the manufacturer, we adopt the full return with partial credit policy proposed in Pasternack (1985) and find the channel's optimal order quantity,  $Q_T$ , satisfies  $F(Q_T) = (p - c - s)/p$ . In addition, the retailer's optimal order quantity,  $Q_R$ , satisfies  $F(Q_R) = (p - c - w)/(p - r)$ . Letting  $r(c)$  denote the buyback price as a function of the retail cost, it is easy to see that setting  $r(c)$  as in Eq. (1) will coordinate the channel

$$r(c) = \frac{p(w-s)}{p-c-s} \quad (1)$$

Namely, under (1) we have  $Q_T = Q_R$ . (1) plays a pivotal role in achieving channel optimality in a decentralized system; as such we refer to it as the *channel-optimal buyback price*.

When the channel is coordinated, we denote the optimal channel order quantity as  $Q^F(c)$ . In this case, the coordinated channel profit is expressed as a type of newsvendor profit function

$$\Pi_T^F(c) = (p-s-c)Q^F(c) - p \int_0^{Q^F(c)} F(x) dx \quad (2)$$

where  $Q^F(c) = F^{-1} \left[ \frac{p-s-c}{p} \right]$ .

If the value of  $w$  is fixed, we state the following proposition:

**Proposition 1.** *With the wholesale price fixed, we have*

- $r(c) < w + c$ ;
- there is  $\bar{c}$ , such that if  $c > \bar{c}$  then  $r(c) > w$ ;
- $\Pi_T^F(c)$  decreases with respect to  $c$ .

All the proofs are in the appendix. Proposition 1(a) says the buyback price is bounded by the sum of the wholesale price and the manufacturer cost, which prohibits the retailer from profiting by buying products from the manufacturer but then returning them immediately. However, Proposition 1(b) implies there are retail costs for which the manufacturer is willing to offer a buyback price that will generate negative margins if returned. The reason behind such willingness is that although the manufacturer loses profit from those returned units for certain materialized demand, he still gains from other materialized demand; therefore the profit is higher in expectation. Finally, it is not surprising that total profit decreases as retail cost increases.

It is clear that the feasible region for the retail cost is  $[0, p - s]$  in the centralized supply chain whereas it is  $[0, p - w]$  in the

decentralized supply chain. When the decentralized supply chain is coordinated, the channel profit is divided into  $\Pi_M^F(c) = \frac{w-s}{p-c-s} \Pi_T^F(c)$  for the manufacturer and  $\Pi_R^F(c) = \left[1 - \frac{w-s}{p-c-s}\right] \Pi_T^F(c)$  for the retailer (Cachon 2003). With such a profit split, setting a wholesale price  $w=p-c$  allows the manufacturer to absorb all the channel profit. However, such a case is not rational because the retailer will refuse to participate. In order for the manufacturer to determine the optimal wholesale price, we define  $w$  as  $w(c)$  in general. Therefore we have a pair of decision variables,  $w(c)$  and  $r(c)$ . If we rephrase (1) as

$$r(c) = \frac{p(w(c)-s)}{p-c-s} \quad (1')$$

the channel is coordinated for any feasible  $c$ .

In order to determine the feasible  $c$ , we propose the individual rationality conditions:

**Assumption 1.** *The retailer will not participate in the transaction without earning a reservation profit  $\Pi_R^-$ ; meanwhile, the manufacturer will not participate without earning a reservation profit  $\Pi_M^-$ .*

In general,  $\Pi_R^-$  and  $\Pi_M^-$  will of course be nonzero. Assumption 1 implies the channel profit must be at least  $\Pi_M^- + \Pi_R^-$  in order for a transaction to occur. In order to have a non-trivial problem, we assume  $\Pi_T^F(0) > \Pi_M^- + \Pi_R^-$  throughout this paper. Since  $\Pi_T^F(c)$  is decreasing and  $\Pi_T^F(p-s) = 0$ , we can define the following threshold of the retail cost:

**Definition 1.** Define a cutoff point  $c^* < p-s$  for  $c$  by  $\Pi_T^F(c^*) = \Pi_M^- + \Pi_R^-$ .

The feasible region  $c \in [0, c^*]$  defines the maximal region where a feasible transaction can be consummated from the channel perspective. In the full-information setting, the manufacturer will set the wholesale price such that the retailer's profitability constraint is binding. The associated wholesale price,  $w_0^*(c)$ , is found by

$$w_0^*(c) = (p-c) - (p-c-s) \frac{\Pi_R^-}{\Pi_T^F(c)}$$

From the above we can deduce that the retailer always earns  $\Pi_R^-$  while the manufacturer extracts the rest of the channel profit.

Under full information, employing an individual rationality constraint on each party, we have proposed a mechanism to determine the wholesale price and buyback price such that the channel is coordinated while maximizing the manufacturer's profit.

### 3. Wholesale-buyback contracts under asymmetric information

In the full-information case, the manufacturer knows the transaction will not take place if the retail cost is above  $c^*$ . So we assume in the asymmetric information case the manufacturer's estimate of the retail cost is in the region  $[D, U] \subset [0, c^*]$ , bounded by an upper bound  $U$  and a lower bound  $D$ , and he estimates the retailer cost has a probability distribution of  $h(c)$  in the region of  $[D, U]$  with the cumulative distribution function of  $H(c)$ . We also assume the retailer's true cost always falls into this region.

With the two instruments of wholesale and buyback prices, the following sequence of events unfolds in the presence of asymmetric retail cost information. First, the manufacturer estimates the retailer's cost; i.e., the OEM can bound the retail cost and likely can estimate it within a relatively narrow range. Second, considering the manufacturer's estimate of the retail cost, the manufacturer offers a contract menu consisting of the wholesale price and the buyback price. Third, the retailer, with full knowledge of the retail cost, accepts a set of wholesale and buyback prices from the menu

and then decides the order quantity. Fourth, the manufacturer delivers this quantity to the retailer. Fifth, the end-consumer demand is realized and the retailer fulfills as much demand as possible, returning any unsold units to the manufacturer.

#### 3.1. Channel coordination under asymmetric information

We will find that an incentive-compatible contract menu consisting of  $\{w(c), r(c)\}$  with the channel-optimal buyback price in (1') can coordinate the channel under asymmetric information, and therefore a channel-optimal solution is achieved in  $c \in [0, c^*]$ . If the retail cost is  $c$  but the retailer accepts the contract  $\{w(c'), r(c')\}$  from the manufacturer, the retailer will determine the optimal order quantity by solving

$$\Pi_R^A(w(c')|c) = \max_{Q(w(c')|c)} (p-w(c')-c)Q(w(c')|c) - (p-r(c')) \int_0^{Q(w(c')|c)} F(x)dx \quad (3)$$

where relation (1') has been inserted in Eq. (3) and therefore we only need to determine the optimal  $w(c)$ . The solution to the above problem will yield the optimal order quantity,  $Q(w(c')|c)$ , as a function of  $w(c')$ . Under this contract, the manufacturer's profit is

$$\Pi_M^A(w(c')|c) = (w(c')-s)Q(w(c')|c) - r(c') \int_0^{Q(w(c')|c)} F(x)dx$$

Therefore, the manufacturer needs to set the optimal wholesale price, followed by the buyback price given in Eq. (1'), by solving the following:

$$\max_{w(c')} \Pi_M^A(w(c')|c) = (w(c')-s)Q(w(c')|c) - r(c') \int_0^{Q(w(c')|c)} F(x)dx$$

where  $c'$  is estimated by the manufacturer while  $c$  is known by the retailer.

In this asymmetric information case, the revelation principle (Laffont and Tirole 1993) states that if there is an optimal contract for the manufacturer, then there exists an optimal contract under which the retailer will truthfully reveal her cost. Thus we are only interested in contracts based on the revelation mechanism. From Eq. (3), the manufacturer can predict how a retailer with cost  $c$  will behave, and therefore what contract she will accept. The manufacturer can construct a mapping between  $c$  and  $c'$  from Eq. (3). The revelation principle is interpreted by the following incentive-compatible condition

$$\Pi_R^A(w(c')|c) < \Pi_R^A(w(c)|c) \quad \forall c, c' \quad (4)$$

Since Eq. (4) will enforce the retailer to tell the truth of her cost while (1') can coordinate the channel, we have:

**Theorem 1.** *Solving Eq. (4) with (1') will coordinate the channel in the asymmetric information case for any  $[D, U]$ , a subset of  $[0, c^*]$ , which gives the following equation in this region for  $w(c)$*

$$\frac{dw(c)}{dc} = (w(c)-s) \frac{p \int_0^{Q^F(c)} F(x)dx}{(p-c-s)\Pi_T^F(c)} \quad (5)$$

Taking into account the individual rationality conditions, we can find if  $U < c^*$ , there are a class of solutions to Eq. (5) which only differ in an initial condition. If  $U = c^*$ , there is only one unique solution to Eq. (5) whereby the channel is coordinated as in the full information case. We will assume  $[D, U]$  to be general in the following.

#### 3.2. Buyback contract under channel optimality

From Theorem 1, a follow-up question is how much profitability the manufacturer can extract from a channel-coordinated contract

in the asymmetric information case. The answer to this question depends on the choice of the right contract range of the retailer cost.

Rather than designing the incentive-compatible contract in the full range of  $[D, U]$ , we will first restrict the contract to  $c \in [D, \bar{U}]$  where  $\bar{U} \leq U$ , and then find the optimal  $\bar{U}$ . The contract is void if  $c \in [\bar{U}, U]$ . This is the cutoff policy introduced in the literature (Corbett et al., 2004; Ha, 2001). The optimality of such a cutoff policy will be demonstrated after solving the contract.

According to this cutoff policy, we develop a contract menu under asymmetric information, labeled as Problem 1.

**Problem 1.** Contract under channel optimality

$$\max_{\bar{U} \in [D, U]} \int_D^{\bar{U}} \max_{w(c)} \Pi_M^A(w(c)|c) h(c) dc \tag{6}$$

Subject to

$$\frac{dw(c)}{dc} = (w(c)-s) \frac{p \int_0^{Q^F(c)} F(x) dx}{(p-c-s) \Pi_T^F(c)} \quad c \in [D, \bar{U}] \tag{7}$$

$$\Pi_R^A(w(c)|c) \geq \Pi_R^- \quad c \in [D, \bar{U}] \tag{8}$$

$$\Pi_M^A(w(c)|c) \geq \Pi_M^- \quad c \in [D, \bar{U}] \tag{9}$$

Here the term ‘‘channel optimality’’ is a loose meaning since the channel profit is equal to that in the full information case only in the region  $c \in [D, \bar{U}]$  instead of the full range of  $c \in [D, U]$ . The objective in Eq. (6) is to optimize the manufacturer’s expected profit under the condition that the channel is coordinated for any  $c \in [D, \bar{U}]$  from Eq. (7), which is the repetition of Eq. (5).

To solve Problem 1, we first solve it with fixed  $\bar{U}$ . We start with solving (7) and yield

$$w(c) = s + (w(D)-s)e^{L(c)} \tag{10}$$

where

$$L(c) = \int_D^c dc' \frac{p \int_0^{Q^F(c')} F(x) dx}{(p-c'-s) \Pi_T^F(c')}$$

and  $w(D)$  is the initial condition to be determined. Since a pair of  $\{w(c), r(c)\}$  can coordinate the channel with retail cost  $c$  in  $c \in [D, \bar{U}]$ , the resulting manufacturer and retailer profits are

$$\Pi_M^A(w(c)|c) = \frac{w(c)-s}{p-s-c} \Pi_T^F(c) \tag{11}$$

$$\Pi_R^A(w(c)|c) = \left[ 1 - \frac{w(c)-s}{p-s-c} \right] \Pi_T^F(c) \tag{12}$$

**Proposition 2.** The optimal solution to Problem 1 is to set  $\Pi_R^A(w(\bar{U})|\bar{U}) = \Pi_R^-$ . Therefore, in the region  $c \in [D, \bar{U}]$ , the manufacturer earns a constant profit which is a function of  $\bar{U}$  and decreasing in  $\bar{U}$ , while the retailer extracts the rest of channel profit which is decreasing in  $c$ .

Setting  $\Pi_R^A(w(\bar{U})|\bar{U}) = \Pi_R^-$  yields the optimal  $w(D)$ , denoted as  $w_1^*(D)$

$$w_1^*(D) = s + (p-s-\bar{U}) \left[ 1 - \frac{\Pi_R^-}{\Pi_T^F(\bar{U})} \right] e^{-L(\bar{U})} \tag{13}$$

Proposition 2 has demonstrated the cutoff policy is valid and optimal, since for  $c > \bar{U}$  the retailer’s individual rationality condition is violated and thus no contract is signed.

From Eq. (13), we have the optimal  $w(c)$  for any  $c \in [D, \bar{U}]$ , denoted by  $w_1^*(c)$ , as

$$w_1^*(c) = s + (p-s-\bar{U}) \left[ 1 - \frac{\Pi_R^-}{\Pi_T^F(\bar{U})} \right] e^{-L(\bar{U})-L(c)} \tag{14}$$

when the buyback price is set as  $r_1^*(c) = p(w_1^*(c)-s)/(p-c-s)$ .

**Corollary 1.** The contract menu  $\{w(c), r(c)\}$  for any fixed  $\bar{U}$  is independent of the retail cost distribution,  $h(c)$ .

While it is self-evident, Corollary 1 implies the contract from Problem 1 is robust as it does not depend on the information of the retail cost distribution.

We need to determine the optimal cutoff point  $\bar{U}$ . If  $\bar{U} = U$  then the manufacturer’s materialized profit is the least for any  $c \in [D, U]$  from Proposition 2, when the contract is non-trivial for the full range of  $c \in [D, U]$ . In this case, the channel achieves the optimality as in the full information case. If we reduce  $\bar{U}$ , then we have a larger value of the manufacturer’s profit for any  $c \in [D, \bar{U}]$ . By varying  $\bar{U}$  in the range of  $[D, U]$ , we then find the optimal  $\bar{U}$  to maximize the manufacturer’s profit. The optimal cutoff point  $\bar{U}$  is found by solving the following one-dimensional optimization problem

$$\begin{aligned} \max_{D \leq \bar{U} \leq U} G(\bar{U}) &= \int_D^{\bar{U}} \max_{w(c)} \Pi_M^A(w(c)|c) h(c) dc \\ &= (p-s-\bar{U}) \left[ 1 - \frac{\Pi_R^-}{\Pi_T^F(\bar{U})} \right] \int_D^{\bar{U}} \frac{\Pi_T^F(c) e^{-L(c)-L(\bar{U})}}{p-c-s} h(c) dc \end{aligned} \tag{15}$$

In the above equation, (11) and (14) are applied in the second equality.

The concavity or convexity of  $G(\bar{U})$  is not clear; in general, it could be non-concave and non-convex. Since Eq. (15) is a one-dimensional problem, it can be solved by a line search. After finding the optimal cutoff point of  $\bar{U}$ , denoted as  $\bar{U}_1^*$ , we can determine the wholesale price and the profit allocation between the two parties for  $c \in [D, \bar{U}_1^*]$ .

3.3. Buyback contract without channel optimality

In this section, we study a wholesale-buyback contract to maximize the manufacturer’s profit under asymmetric cost information without considering the channel performance. Informally speaking, the manufacturer will offer a contract menu of wholesale and buyback prices without condition (1’). We again employ the revelation principle with the cutoff policy. With the manufacturer estimating the retail cost in the region  $[D, U] \subset [0, c^*]$ , we assume the manufacturer issues contracts only if the retail cost is less than a threshold  $\bar{U} \in [D, U]$  while the retailer will at least receive her minimal expected profit in this region. Such a cutoff policy will be proved to be optimal again after the whole contract is solved. If the retailer accepts the contract of  $w(c'), r(c')$  with her real cost value of  $c$ , the retailer solves

$$\begin{aligned} \Pi_R^A(w(c'), r(c')|c) &= \max_{Q(w(c'), r(c')|c)} (p-w(c')-c) Q(w(c'), r(c')|c) \\ &\quad - (p-r(c')) \int_0^{Q(w(c'), r(c')|c)} F(x) dx \end{aligned}$$

and yields the optimal  $Q(w(c'), r(c')|c)$ . The manufacturer’s profit is then given by

$$\Pi_M^A(w(c'), r(c')|c) = (w(c')-s) Q(w(c'), r(c')|c) - r(c') \int_0^{Q(w(c'), r(c')|c)} F(x) dx$$

The manufacturer can now find a menu of wholesale and buyback prices by solving the following incentive-compatible truth-telling optimization problem:

**Problem 2.** Manufacturer-optimal contract

$$\max_{D \leq \bar{U} \leq U} \int_D^{\bar{U}} \max_{w(c), r(c)} \Pi_M^A(w(c), r(c)|c) h(c) dc \tag{16}$$

subject to

$$\Pi_R^A(w(c'), r(c')|c) \leq \Pi_R^A(w(c), r(c)|c) \quad D \leq c, c' \leq \bar{U} \tag{17}$$

$$\Pi_R^A(w(c), r(c)|c) \geq \Pi_R^- \quad D \leq c \leq \bar{U} \quad (18)$$

$$\Pi_M^A(w(c), r(c)|c) \geq \Pi_M^- \quad D \leq c \leq \bar{U} \quad (19)$$

Though the above contract is typical in the principal-agent literature, we are questioning whether this must be the best that the manufacturer should offer, in particular if the leader (manufacturer) in the channel is also sensitive to the channel performance. We will address this question after solving Problem 2 and then compare it with the contract in Problem 1.

We demonstrate in the Appendix that the wholesale-buyback contract in Problem 2 is equivalent to the nonlinear-pricing contract in the literature. To solve it completely, we need the following mild assumption,

**Assumption 2.** Defining  $z(c) = H(c)/h(c)$ , the retailer cost distribution satisfies  $z'(c) = dz(c)/dc > 0$ .

This assumption means the distribution of the retail cost is decreasing reversed hazard rate, which is true for many distribution functions (Corbett and De Groote 2000).

Under Assumption 2, we have

**Proposition 3.** The optimal solutions of  $(w(c), r(c))$  for  $c \in [D, \bar{U}_2^*]$  in Problem 2, denoted as  $(w_2^*(c), r_2^*(c))$ , satisfy

$$(a) \quad r_2^*(c) = \frac{p(w_2^*(c) - z(c) - s)}{p - c - z(c) - s}$$

$$(b) \quad \frac{dw_2^*(c)}{dr_2^*(c)} = \frac{\int_0^{q^*(c)} F(x)dx}{q^*(c)}$$

where

$$(c) \quad q^*(c) = F^{-1} \left[ \frac{p - c - z(c) - s}{p} \right]$$

$$(d) \quad \bar{U}_2^* = \min(U, c_\alpha, c_\beta)$$

$$(e) \quad c_\alpha + z(c_\alpha) + s = p$$

$$(f) \quad (p - c_\beta - z(c_\beta) - s)q^*(c_\beta) - p \int_0^{q^*(c_\beta)} F(x)dx = \Pi_M^- + \Pi_R^-$$

The relation between  $w_2^*(c)$  and  $r_2^*(c)$  in Proposition 3(a) resembles that in Eq. (1). In the proof of Proposition 3, we also show that both the manufacturer and the retailer's profits from the contract menu decrease with respect to  $c$ , and therefore the cutoff policy is optimal. Moreover, these profit functions are only functions of the optimal order quantity,  $q^*(c)$ . We therefore first derive the optimal  $q^*(c)$  and  $\bar{U}_2^*$ , and then the profits for the manufacturer and the retailer.

We are also interested in the contract instruments for Problem 2. To this end, we first define

$$g(c) = \frac{\int_0^{q^*(c)} F(x)dx}{q^*(c)}$$

$$A(c) = -\frac{1 + z(c)}{p(1 - g(c)) - c - z(c) - s}$$

$$B(c) = -\frac{pz'(c)}{p(1 - g(c)) - c - z(c) - s}$$

Then, we have:

**Corollary 2.** The optimal solutions to  $(w_2^*(c), r_2^*(c))$  are solved as

$$r_2^*(c) = \frac{\int_D^c e^{\int_D^{c'} A(c'')dc''} B(c')dc' + \beta}{e^{\int_D^c A(c'')dc''}} \quad (20)$$

$$w_2^*(c) = z(c) + s + \frac{p - c - z(c) - s}{p} r_2^*(c) \quad (21)$$

where the initial condition,  $\beta$ , could be given arbitrarily as long as  $r_2^*(c)$  is non-negative. With the relation of (21), this constant will not affect the order quantity and the profits in the system.

#### 4. Numerical results

In this numerical experiment, we will compare the performance of the contract under channel optimality (denoted as COC) from Section 3.2, and the contract without channel optimality (denoted as MOC) from Section 3.3. In particular, we want to see the difference between the cutoff points and the profits of each party in the two contracts. We will focus on the effects of reservation profits as well as the manufacturer's estimate of the retailer's retail cost.

Throughout this section, the demand is assumed to be a normal distribution Normal (200, 50) and the retail cost is uniformly distributed in  $[D, U]$ . The end-customer selling price is fixed as  $p = \$210$  throughout this section, while the manufacturing cost is first chosen as  $s = \$50$  but will be varied later. Therefore, if the channel is centralized, the channel profit is simply given by Eq. (2) and the feasible region of  $c$  is  $[\$0, 160]$ . If the channel is decentralized with full information, the cutoff point  $c^*$  for the feasible region of  $c$  is determined by the retailer and manufacturer reservation profits. In the asymmetric information case with the manufacturer's estimate of the retail cost as  $[D, U]$ , we solve COC and MOC. In Fig. 1, we depict the profits of the manufacturer, the retailer and the channel under both contracts when  $\Pi_M^- = \$5000$ ,  $\Pi_R^- = \$3000$  and  $[D, U] = [\$0, 90]$ . It is found the cutoff point  $c^* = \$102.5$  in the full information case,  $\bar{U}_1^* = \$68$  in COC, and  $\bar{U}_2^* = \$51$  in MOC.

From Fig. 1, the regions of the retail cost for non-trivial COC and MOC are less than  $[0, c^*]$ , while the feasible region in MOC is smaller than that in COC. It is also observed that the retailer's realized profit for a given  $c$  in COC is always higher than that in MOC while the manufacturer's realized profit for a given  $c$  in MOC is higher than that in COC, except that in the region  $[\bar{U}_1^*, \bar{U}_2^*]$  where the MOC contract does not exist. The channel profit performs very closely in the two contracts in  $[0, \bar{U}_1^*]$ .

In Fig. 2, we depict the contract instruments of both COC and MOC.

From Fig. 2, the wholesale prices in the COC and MOC are quite close; however, the buyback prices differ dramatically with the retail cost increasing. For COC, the manufacturer has to pay higher buyback price in order to achieve channel coordination; the return is a wider region of the retailer cost that warrants an effective contract. In the MOC, the manufacturer is overall conservative since the channel performance is beyond his concern. As a note, the constant,  $\beta$ , in Eq. (20), is chosen such that the buyback price at the cutoff point,  $\bar{U}_2^*$ , is zero. Indeed, the buyback price in Eq. (20) is decreasing in  $c$  in this case.

With all the other problem parameters unchanged, we will now vary  $(\Pi_M^-, \Pi_R^-)$  and  $[D, U]$ . In Fig. 3, we have changed  $[D, U]$  to a much narrower region,  $[D, U] = [\$10, 40]$ . In this case, the cutoff points in COC and MOC are the same, namely,  $\bar{U}_1^* = \bar{U}_2^* = \$40$ ; the profits of the channel, manufacturer and retailer in MOC are very close to their counterparts in the COC for almost all of  $c \in [D, U]$ .

In order to further understand the effects of the manufacturer's estimate of the retail cost, we choose another set of  $[D, U]$  as  $[\$60, 80]$  while keeping the reservation profits of  $\Pi_M^- = \$5000, \Pi_R^- = \$3000$ . In this case, the optimal cutoff point  $\bar{U}_1^*$  in COC is solved to be  $\$70$ , while the optimal cutoff point in MOC is  $\bar{U}_2^* = \$80$ . The profits of the manufacturer, the retailer, and the channel are depicted in Fig. 4. From Fig. 4, the channel profits in COC and MOC are almost identical in the region of  $c \in [D, \bar{U}_1^*]$ , while in  $c \in [\bar{U}_1^*, \bar{U}_2^*]$  the channel only achieves profitability in MOC. The manufacturer's realized profit in  $c \in [D, \bar{U}_1^*]$  in COC is larger than that in MOC; however, he earns a profit in the region

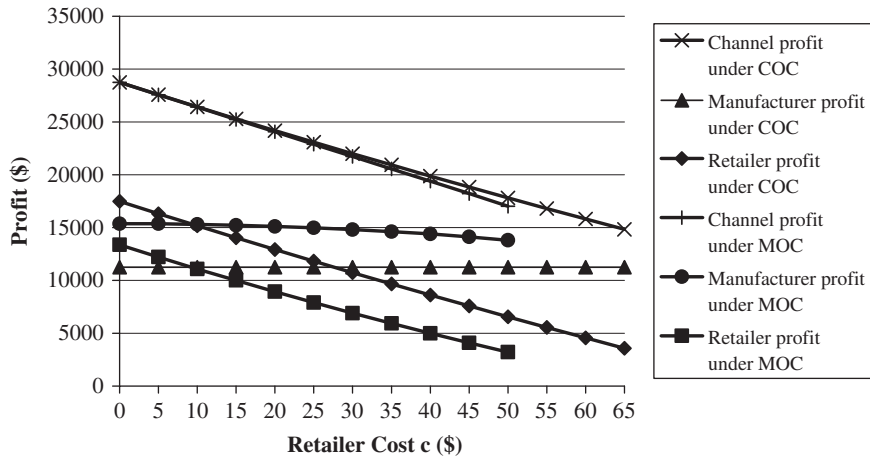


Fig. 1. Profit allocation with  $\Pi_M^- = \$5000, \Pi_R^- = \$3000, [D, U] = \$[0, 90], s = \$50$ .

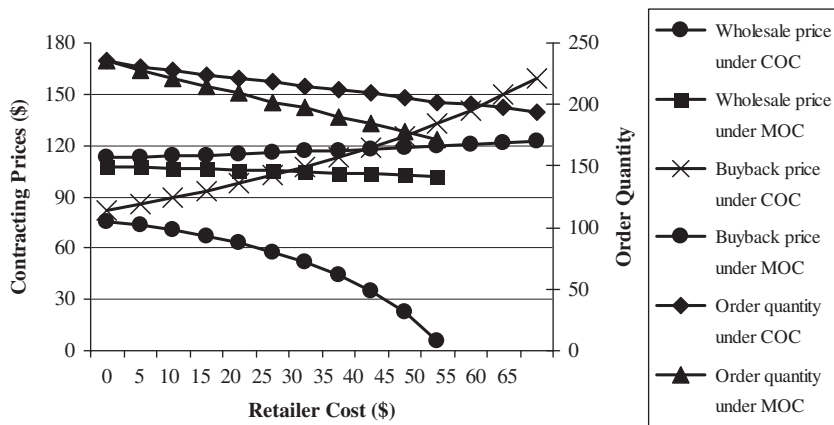


Fig. 2. Contract instruments for  $\Pi_M^- = \$5000, \Pi_R^- = \$3000, [D, U] = \$[0, 90], s = \$50$ .

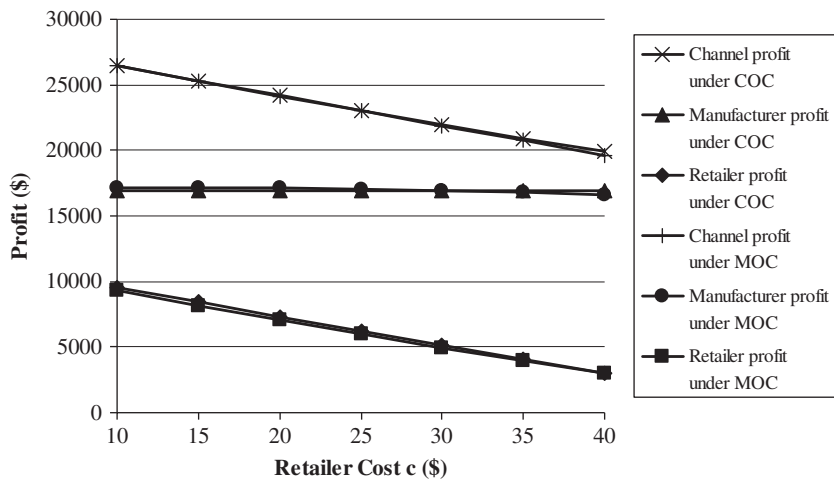


Fig. 3. Profit allocation with  $\Pi_M^- = \$5000, \Pi_R^- = \$3000, [D, U] = \$[10, 40], s = \$50$ .

of  $[\bar{U}_1, \bar{U}_2^*]$  in MOC. On the other hand, the retailer's realized profit for a given  $c$  in MOC is larger than the counterpart in COC.

With Figs. 1–4 as illustrative examples, we want to broaden our understanding of COC and MOC through a larger number of examples. We have chosen three cases of  $(\Pi_M^-, \Pi_R^-)$ , corresponding to high, medium, and low individual rationalities for the manufacturer and the retailer. For each pair of  $(\Pi_M^-, \Pi_R^-)$ , we find  $c^*$  and then choose four

segments in the range of  $[0, c^*]$ , which represent the cases that the retail cost spans in broad, low, medium, and high regions, respectively. For each combination of  $(\Pi_M^-, \Pi_R^-)$  and  $[D, U]$ , we find the cutoff points and expected profits of the channel, the manufacturer and the retailer for both contracts. In Table 1, we have summarized the results for the 20 tested cases. The last column in Table 1 is for the percentage changes of the profits from the MOC to the COC for different entities.

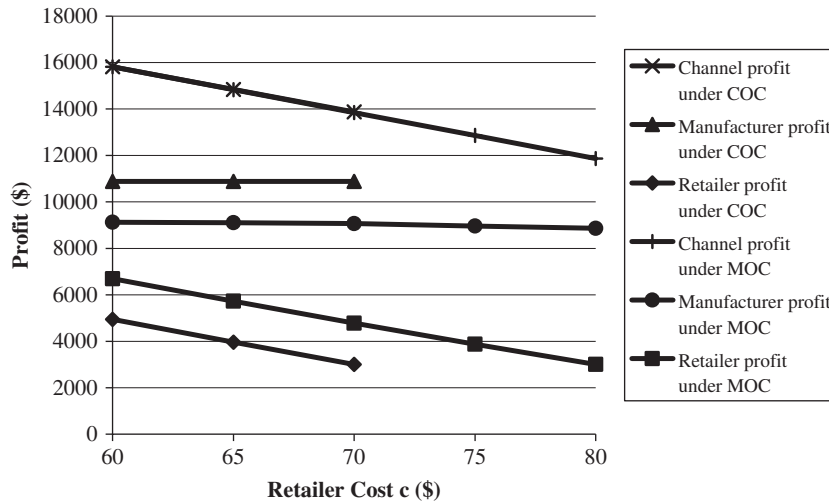


Fig. 4. Profit allocation with  $\Pi_M^- = \$5000, \Pi_R^- = \$3000, [D, U] = \$[60, 80], s = \$50$ .

Table 1  
Summary of tested cases ( $p = \$210, s = \$50$ ).

Case	$\Pi_M^-, \Pi_R^-$ \$	$c^*$ \$	$(D, U)$ \$	$\bar{U}_1^*$ \$	COC			$\bar{U}_2^*$ \$	MOC			% change of profit		
					Manuf profit (\$)	Retailer profit (\$)	System profit (\$)		Manuf profit (\$)	Retailer profit (\$)	System profit (\$)	Manuf	Retailer	System
1	(50, 30)	158.7	(0,140)	81	6818	4720	11,538	79	7253	3629	10,882	-6.0%	30.0%	6.0%
2			(10,50)	50	17,789	4252	22,041	50	17,950	3929	21,879	-0.9%	8.2%	0.7%
3			(60, 100)	82	6380	1155	7535	100	8593	3231	11,824	-25.8%	-64.3%	-36.3%
4	(500, 300)	151.4	(0, 140)	80	6685	4768	11,453	76	7097	3575	10,672	-5.8%	33.3%	7.3%
5			(10,50)	50	17,519	4522	22,041	50	17,680	4199	21,879	-0.9%	7.7%	0.7%
6			(60, 100)	81	1197	6045	7242	100	8323	3500	11,824	-85.6%	72.7%	-38.8%
7	(500, 3000)	130.9	(110, 140)	110	NA	NA	NA	131	2282	1176	3458	NA	NA	NA
8			(0,120)	70	6346	5938	12,284	65	6653	4827	11,480	-4.6%	23.0%	7.0%
9			(10,40)	40	16,879	6228	23,107	40	16,970	6046	23,016	-0.5%	3.0%	0.4%
10	(5000, 300)	119	(50,80)	69	7011	3076	10,087	80	9094	5672	14,765	-22.9%	-45.8%	-31.7%
11			(90, 120)	80.5	NA	NA	NA	95	2543	3146	5689	NA	NA	NA
12			(0, 100)	80	10,399	7416	17,815	59	10,565	3828	14,393	-1.6%	93.7%	23.8%
13	(5000, 3000)	102.5	(10,40)	40	19,579	3528	23,107	40	19,670	3346	23,016	-0.5%	5.5%	0.4%
14			(40, 70)	70	13,579	3255	16,834	70	13,670	3070	16,740	-0.7%	6.0%	0.6%
15			(70, 100)	80	4222	486	4708	90	7477	1941	9418	-43.5%	-75.0%	-50.0%
16	(5000, 3000)	102.5	(0, 90)	68	8509	7557	16,066	51	8447	4508	12,955	0.7%	67.6%	24.0%
17			(10,40)	40	16,879	6228	23,107	40	16,970	6046	23,016	-0.5%	3.0%	0.4%
18			(40, 70)	70	10,879	5955	16,834	70	10,970	5770	16,740	-0.8%	3.2%	0.6%
19	(5000, 3000)	102.5	(70,90)	NA	NA	NA	NA	86	6409	3611	10,020	NA	NA	NA
20														

Table 2  
More tested cases ( $p = \$210, s = \$100$ ).

Case	$\Pi_M^-, \Pi_R^-$ \$	$c^*$ \$	$(D, U)$ \$	COC			MOC			% change of profit		
				Manuf profit (\$)	Retailer profit (\$)	System profit (\$)	Manuf profit (\$)	Retailer profit (\$)	System profit (\$)	Manuf	Retailer	System
21	(50, 30)	108.6	(0,108)	4685	1701	6387	4049	2027	6077	15.7%	-16.1%	5.1%
22			(10,40)	10,834	2119	12,954	10,251	2597	12,849	5.7%	-18.4%	0.8%
23			(40,70)	2375	1003	3378	5253	2192	7445	-54.8%	-54.2%	-54.6%
24	(500, 300)	101.0	(70, 100)	NA	NA	NA	1530	768	2298	NA	NA	NA
25			(0,100)	5119	1701	6821	4222	2136	6359	21.2%	-20.4%	7.3%
26			(10,40)	10,546	2407	12,954	9981	2867	12,849	5.7%	-16.0%	0.8%
27	(5000, 3000)	52.5	(40,70)	3638	310	4178	4983	2462	7445	-27.0%	-87.4%	-43.9%
28			(70, 100)	NA	NA	NA	1342	697	2039	NA	NA	NA
29			(0,50)	6284	6009	12,293	5129	2827	7956	22.5%	112.6%	54.5%
30	(5000, 3000)	52.5	(10,30)	12,995	3818	16,813	12,828	3975	16,803	1.3%	-3.9%	0.1%
31			(30,50)	NA	NA	NA	3925	2244	6169	NA	NA	NA

From Table 1, we can draw a number of observations. First, higher retail cost will lead to lower channel, manufacturer and retailer profits in both COC and MOC. Moreover, when this cost is in a very high region, such as in cases 4, 8, 12, and 20, COC becomes void, whereas MOC is still valid; we have used “NA” to denote the cases that no effective contract is signed. This indicates that high retail cost needs high wholesaler price to achieve channel optimality; however, this may break the manufacturer’s rationality condition. Second, if the retail cost is in a region with relatively low value, the channel’s profit in COC is higher than that in MOC, while the manufacturer’s profit in COC is very close to that in MOC. Interestingly, in case 17 the manufacturer’s profit in COC even could be higher than that in MOC, indicating that the channel optimality and individual optimality could be consistent. This might look surprising at the first glance as we may regard the COC as a special case of MOC with an additional constraint. As a matter of fact, with the cutoff point policy in both contracts this is not true anymore. We have observed in Case 17 the cutoff point in COC is higher than MOC.

While Case 17 looks unique in Table 1, it may happen more frequently if we vary the product price,  $p$ , and cost,  $s$ . In fact, the difference between  $p$  and  $s$  automatically restrict the region for the retailer cost from Proposition 1; therefore, we fix  $p$  but change  $s$ . In Table 2, we list the profits of the manufacturer, retailer, and system, as well as their percentage relations by letting  $s = \$100$  while varying  $\Pi_M$ ,  $\Pi_R$  and  $(D, U)$  in the meantime. From this table, in cases 21, 22, 25, 26, 29, 30, both the manufacturer and the system perform better in the COC than in the MOC; in cases 23, 24, 27, 28, 31, in which the retailer costs fall into high segments of the feasible regions, the COC is a choice from each party and the system. Comparing Table 2 with Table 1, it indicates that if the product margin is low, using COC cannot only coordinate the channel but also yield high profit for the manufacturer if the retailer cost is not estimated in a high-value region.

In practice, the retailer cost should not be too high so the COC should be valuable in many cases. Therefore, it pays for the OEM to understand the trade-off between the two contracts under various conditions. There are also several situations where the manufacturer and the channel are in the win-win situation with specific data, which supports the hypothesis raised in the introduction. This should also happen to more scenarios if we change other problem conditions in the problem, for example, the demand profile. Overall, there is no simple way to determine a threshold where the retailer cost dictates the use of COC versus MOC. That said,

**Appendix**

**Proof of Proposition 1.**

(1) From Eq. (1) in the text, we have

$$r(c) - w - c = \frac{p(w-s)}{p-c-s} - w - c = -\frac{(p-w-c)(c+s)}{p-c-s} < 0.$$

(2) It is easy to check if  $p-w > c > \bar{c} = \frac{s(p-w)}{w}$  where  $s < w$  by assumption, then we have  $r(c) > w$ .

(3)  $\frac{\partial \Pi_T^F(c)}{\partial c} = -Q^F(c) < 0.$

**Proof of Theorem 1.** We prove condition (5) is necessary and sufficient to induce the retailer to tell her cost truthfully.

(1) Necessary condition

We first prove that if the extreme value for  $w(c')$  to maximize  $\Pi_R^A(w(c')|c)$  is  $w(c)$ , then  $w(c)$  satisfies Eq. (5). This is proved starting with Eq. (3), from which we can derive the optimal value of  $Q(w(c')|c)$  satisfies

$$(p-w(c')-c)-(p-r(c'))F(Q(w(c')|c))=0. \tag{A1}$$

Therefore we can derive from some straightforward algebra that

$$\frac{\partial \Pi_R^A(w(c')|c)}{\partial c'} \Big|_{c'=c} = -\frac{dw(c)}{dc} Q(w(c)|c) + (p-w(c)-c) \frac{\partial Q(w(c')|c)}{\partial c'} \Big|_{c'=c} + \frac{\partial r(c')}{\partial c'} \Big|_{c'=c} \int_0^{Q(w(c)|c)} F(x) dx$$

solving the two contracts in this paper will tell which one is more suitable from the perspective of the manufacturer and the channel since the manufacturer is the Stackelberg leader.

**5. Conclusions**

We have examined the performance of wholesale-buyback contracts in the presence of retail cost. When this cost information is known to the manufacturer, a buyback contract can fully coordinate the channel. We decide the wholesale price taking into account the individual rationality constraints for both the manufacturer and the retailer.

When the retail cost is private information, it is not easy to achieve channel optimality. However, in this paper, we propose an incentive-compatible wholesale-buyback contract that can achieve channel optimality. Starting with such a coordinated channel and asymmetric information, we propose a contract to maximize the manufacturer’s profit. We then propose a new contract to maximize the manufacturer’s profit, and solve it by mapping it onto a nonlinear-pricing contract. The difference between these two contracts lies in a specific wholesale-buyback condition which has been derived in the full information case that can coordinate the channel. From the practical perspective, we are particularly interested in implementing the results in this paper in real world such as the motivating problem in the introduction.

There are several logical areas to extend this research. First, we are interested in a more complicated contracting mechanism such that not only the channel is coordinated but also the manufacturer is always better off than in the MOC. Second, a natural evolution of this research will be to integrate both asymmetric cost information and asymmetric demand information into the problem setting. This question arises naturally as we have observed substantial research towards either of the two, but not much towards both.

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$$-(p-r(c))F(Q(w(c)|c))\frac{\partial Q(w(c)|c)}{\partial c'}\Big|_{c'=c} = -\frac{dw(c)}{dc}Q(w(c)|c) + \left[\frac{p}{p-c-s}\frac{dw(c)}{dc} + \frac{p(w(c)-s)}{(p-c-s)^2}\right] \int_0^{Q(w(c)|c)} F(x)dx$$

where we have used (A1) at  $c'=c$  and

$$\frac{\partial r(c')}{\partial c'}\Big|_{c'=c} = \frac{p}{p-c-s}\frac{dw(c)}{dc} + \frac{p(w(c)-s)}{(p-c-s)^2}$$

The incentive-compatible condition Eq. (4) implies  $\frac{\partial \Pi_R^A(w(c)|c)}{\partial c'}\Big|_{c'=c} = 0$  and yields

$$\frac{dw(c)}{dc} = \frac{p(w(c)-s) \int_0^{Q^F(c)} F(x)dx}{(p-c-s)[(p-c-s)Q^F(c) - p \int_0^{Q^F(c)} F(x)dx]} = \frac{p(w(c)-s) \int_0^{Q^F(c)} F(x)dx}{(p-c-s)\Pi_7^F(c)}$$

where we have replaced the solution to (A1) at  $c'=c$  by  $Q^F(c)$  from Section 2. It concludes that Eq. (5) dictates the extreme path for  $w(c)$ .

(2) Sufficient condition

The proof of the sufficient condition follows Laffont and Tirole (1993), pp121. We prove Eq. (5) is sufficient by showing  $\Pi_R^A(w(c')|c) < \Pi_R^A(w(c)|c)$  for any  $c' \neq c$  under this condition. If this is not true, there is  $c' \neq c$  such that  $\Pi_R^A(w(c')|c) > \Pi_R^A(w(c)|c)$ . We first prove this causes contradiction assuming  $c' > c$ .

From the proof of the necessary condition, Eq. (5) is from  $D_1 \Pi_R^A(w(c')|c)\Big|_{c'=c} = 0$ , where  $D_1$  is the differentiation with respect to the first variable,  $c'$ . That said,  $D_1 \Pi_R^A(w(x)|x) = 0, \forall x$  under (5). If  $\Pi_R^A(w(c')|c) > \Pi_R^A(w(c)|c)$ , then  $\int_c^{c'} D_1 \Pi_R^A(w(x)|c)dx > 0$ , which further implies

$$\int_c^{c'} (D_1 \Pi_R^A(w(x)|c) - D_1 \Pi_R^A(w(x)|x))dx = \int_c^{c'} dx \int_x^c D_{12} \Pi_R^A(w(x)|u)du > 0$$

where  $D_{12} \Pi_R^A(w(x)|u) = (\partial^2 \Pi_R^A(w(x)|u) / \partial x \partial u)$  and the second differentiation is with respect to  $u$ . However

$$\begin{aligned} D_{12} \Pi_R^A(w(x)|u) &= \frac{\partial^2}{\partial x \partial u} \left[ (p-w(x)-u)Q(w(x)|u) + (p-r(x)) \int_0^{Q(w(x)|u)} F(x)dx \right] \\ &= -\frac{\partial Q(w(x)|u)}{\partial x} - \frac{\partial w(x)}{\partial x} \frac{\partial Q(w(x)|u)}{\partial u} + F(Q(w(x)|u)) \frac{\partial r(x)}{\partial x} \frac{\partial Q(w(x)|u)}{\partial u} - (p-r(x))f(Q(x|u)) \frac{\partial Q(w(x)|u)}{\partial x} \frac{\partial Q(w(x)|u)}{\partial u} \\ &\quad + [(p-w(x)-u) - (p-r(x))F(Q(w(x)|u))] \frac{\partial^2 Q(w(x)|u)}{\partial x \partial u} = -\frac{\partial Q(w(x)|u)}{\partial x} - \frac{\partial w(x)}{\partial x} \frac{\partial Q(w(x)|u)}{\partial u} \\ &\quad + F(Q(w(x)|u)) \frac{\partial r(x)}{\partial x} \frac{\partial Q(w(x)|u)}{\partial u} - (p-r(x))f(Q(w(x)|u)) \frac{\partial Q(w(x)|u)}{\partial x} \frac{\partial Q(w(x)|u)}{\partial u} \end{aligned}$$

where we have employed

$$(p-w(x)-u) - (p-r(x))F(Q(w(x)|u)) = 0. \tag{A2}$$

which repeats Eq. (A1). Differentiating Eq. (A2) with respect to  $u$  yields

$$1 + (p-r(x))f(Q(w(x)|u)) \frac{\partial Q(w(x)|u)}{\partial u} = 0 \tag{A3}$$

Therefore

$$D_{12} \Pi_R^A(w(x)|u) = \frac{\partial Q(w(x)|u)}{\partial u} \left[ -\frac{\partial w(x)}{\partial x} + \frac{\partial r(x)}{\partial x} F(Q(w(x)|u)) \right]$$

We further differentiate Eq. (A2) with respect to  $x$  yielding

$$-\frac{\partial w(x)}{\partial x} + \frac{\partial r(x)}{\partial x} F(Q(w(x)|u)) = (p-r(x))f(Q(w(x)|u)) \frac{\partial Q(w(x)|u)}{\partial x} \tag{A4}$$

Hence

$$D_{12} \Pi_R^A(w(x)|u) = (p-r(x))f(Q(w(x)|u)) \frac{\partial Q(w(x)|u)}{\partial x} \frac{\partial Q(w(x)|u)}{\partial u}$$

Since  $F(Q(w(x)|u)) = (p-w(x)-u)/(p-r(x))$ , we can derive  $\partial Q(w(x)|u)/\partial u = -1/f(Q(w(x)|u))(p-r(x))$  resulting in  $\partial Q(w(x)|u)/\partial u < 0$ . In the next, we prove  $\partial Q(w(x)|u)/\partial x > 0$ , which is achieved by proving  $\partial F(Q(w(x)|u))/\partial x > 0$  due to  $\partial F(Q(w(x)|u))/\partial x = f(Q(w(x)|u))(\partial Q(w(x)|u)/\partial x)$ . In other words, we need to prove that  $(\partial/\partial x)[(p-w(x)-u)/(p-r(x))] > 0$ . However, we can prove by some calculations that

$$\frac{\partial}{\partial x} \left[ \frac{p-w(x)-u}{p-r(x)} \right] = \frac{1}{(p-r(x))^2} \left[ \frac{p(x-u)}{p-x-s} \frac{dw(x)}{dx} + \frac{p(p-w(x)-u)}{(p-x-s)^2} (w(x)-s) \right]$$

Since  $x \in [c, c']$ ,  $x \geq c$ , i.e.,  $u < x$ , and  $dw(x)/dx > 0$  from Eq. (5), we see the first term in the bracket of the right side in the above equation is positive. The second term in the bracket is positive too because of  $w(x) > s$  and  $p > w(x)+u$  from the assumptions in Section 2. Therefore,

$\partial Q(w(x)|u)/\partial x > 0$ , resulting in  $D_{12}\Pi_R^A(w(x)|u) < 0$  and

$$\int_c^{c'} \int_x^{c'} D_{12}\Pi_R^A(w(x)|u)du < 0$$

However, this conclusion is in contradiction to the assumption. If we assume  $c' < c$ , we have the same conclusion. Therefore, condition (5) is sufficient for the incentive compatibility.

**Proof of Proposition 2.** When the retailer truthfully reveals the retail cost information, the channel is coordinated. The manufacturer's profit is

$$\Pi_M^A(w(c)|c) = (w(c)-s)Q^F(c)-r(c) \int_0^{Q^F(c)} F(x)dx$$

where  $Q^F(c) = F^{-1}[(p-w(c)-c)/(p-r(c))]$  while  $w(c)$  and  $r(c)$  satisfy Eqs. (1) and (5). We then have

$$\begin{aligned} \frac{\partial \Pi_M^A(w(c)|c)}{\partial c} &= \frac{dw(c)}{dc} Q^F(c) - \frac{dr(c)}{dc} \int_0^{Q^F(c)} F(x)dx + [(w(c)-s)-r(c)F(Q^F(c))]\frac{\partial Q^F(c)}{\partial c} \\ &= \frac{dw(c)}{dc} Q^F(c) - \frac{dr(c)}{dc} \int_0^{Q^F(c)} F(x)dx + \left[ (w(c)-s)-r(c)\frac{p-w(c)-c}{p-r(c)} \right] \frac{\partial Q^F(c)}{\partial c} = \frac{dw(c)}{dc} Q^F(c) - \frac{dr(c)}{dc} \int_0^{Q^F(c)} F(x)dx \end{aligned}$$

where we have plugged (1') into the second identity. We further use the expressions for  $(dw(c)/dc)$  and  $(dr(c)/dc)$  derived in the proof for Theorem 1 and then have

$$\frac{\partial \Pi_M^A(w(c)|c)}{\partial c} = \frac{dw(c)}{dc} Q^F(c) - \left[ \frac{p}{p-c-s} \frac{dw(c)}{dc} + \frac{p(w(c)-s)}{(p-c-s)^2} \right] \int_0^{Q^F(c)} F(x)dx = \frac{dw(c)}{dc} \frac{\Pi_T^F(c)}{p-c-s} - \frac{p(w(c)-s)}{(p-c-s)^2} \int_0^{Q^F(c)} F(x)dx$$

Therefore,  $\partial \Pi_M^A(w(c)|c)/\partial c = 0$  from Eq. (5), or,  $\Pi_M^A(w(c)|c)$  is constant with respect to  $c$ . Moreover, for  $c \in [D, \bar{U}]$ ,  $\Pi_R^A(w(c)|c) = \Pi_T^F(c) - \Pi_M^A(w(c)|c)$ , so  $\Pi_R^A(w(c)|c)$  is decreasing of  $c$ . The manufacturer will offer the  $w(c)$  to maximize his profit by binding the retailer's profit at  $c = \bar{U}$ . From this condition, the less  $\bar{U}$  is, the higher  $\Pi_M^A(w(c)|c) = \Pi_M^A(w(\bar{U})|\bar{U})$  is. Therefore, Proposition 2 is proved.

**Proof of Proposition 3.** In order to solve Problem 2, we take  $\frac{\partial \Pi_R^A(w(c),r(c)|c)}{\partial c} \Big|_{c=\bar{c}} = 0$  and yield a weak form, or a necessary condition, for (17)

$$-\dot{w}(c)F^{-1}\left[\frac{p-w(c)-c}{p-r(c)}\right] + \dot{r}(c) \int_0^{F^{-1}\left[\frac{p-w(c)-c}{p-r(c)}\right]} F(x)dx = 0 \tag{A5}$$

where  $\dot{w}(c) = (dw(c)/dc)$ ,  $\dot{r}(c) = (dr(c)/dc)$ . With Eq. (A5), we can derive

$$\frac{\partial \Pi_R^A(w(c),r(c)|c)}{\partial c} = -F^{-1}\left[\frac{p-w(c)-c}{p-r(c)}\right] = -Q(w(c),r(c)|c)$$

for which we have the integration form

$$\Pi_R^A(w(c),r(c)|c) = \Pi_R^- + \int_c^{\bar{U}} Q(w(c),r(c)|c)dc \tag{A6}$$

where we have set  $\Pi_R^A(w(\bar{U}),r(\bar{U})|\bar{U}) = \Pi_R^-$ . The fact that  $\Pi_R^A(w(c),r(c)|c)$  decreases with respect to  $c$  from (A6) validates the effectiveness of the cutoff policy from the retailer perspective. Moreover, Eq. (A6) implies that the first-best solution in the full information case can no longer be achieved in this contract.

Now solving Problem 2 is equivalent to solving (16) with (19), (A5) and (A6). Since we are mainly interested in the manufacturer's profits, retailer's profits, and the channel profits, we can circumvent the difficulty in the following way. Given a pair of  $(w(c), r(c))$  for any  $c$ , we have  $Q(w(c),r(c)|c) = F^{-1}[(p-w(c)-c)/p-r(c)]$ , and the channel profit is

$$\Pi_T^A(w(c),r(c)|c) = (p-c-s)Q(w(c),r(c)|c) - p \int_0^{Q(w(c),r(c)|c)} F(x)dx \tag{A7}$$

Since  $\Pi_M^A(w(c),r(c)|c) = \Pi_T^A(w(c),r(c)|c) - \Pi_R^A(w(c),r(c)|c)$ , from (A6) and (A7) we can re-write the manufacturer's profit as

$$\Pi_M^A(w(c),r(c)|c) = (p-c-s)Q(w(c),r(c)|c) - p \int_0^{Q(w(c),r(c)|c)} F(x)dx - \int_c^{\bar{U}} Q(w(c),r(c)|c) - \Pi_R^- \tag{A8}$$

Observing that in (A6), (A7) and (A8) the profit functions are functions of  $Q(w(c),r(c)|c)$  rather than  $w(c),r(c)$  independently, we write  $q(c) = Q(w(c),r(c)|c)$ . We will solve  $(w(c), r(c))$  in Problem 2 by first solving  $q(c)$  in the following problem:

**Problem 3.** Solving for  $q(c)$

$$\max_{D \leq \bar{u} \leq U} \int_D^{\bar{U}} \max_{q(c)} \Pi_M^A(q(c)|c)h(c)dc = \max_{D \leq \bar{u} \leq U} \int_D^{\bar{U}} \max_{q(c)} \left[ (p-c-s)q(c) - p \int_0^{q(c)} F(x)dx - \int_c^{\bar{U}} q(x)dx - \Pi_R^- \right] h(c)dc \tag{A9}$$

$$\text{Subject to } \Pi_M^A(q(c)|c) \geq \Pi_M^- \tag{A10}$$

$$\Pi_R^A(q(c)|c) = \Pi_R^- + \int_c^{\bar{U}} q(x)dx \tag{A11}$$

The solution to Problem 3 is dependent on the distribution of the retail cost. With Assumption 2, the manufacturer's profit in Eq. (A9) could be proved to decrease with respect to  $c$ , validating the cutoff policy from the manufacturer perspective; in other words, the cutoff policy is optimal. We can then derive the optimal value for  $q(c)$  and  $\bar{U}$  as

$$q^*(c) = F^{-1} \left[ \frac{p - c - z(c) - s}{p} \right] \quad c \in [D, \bar{U}_2^*] \quad (\text{A12})$$

$$\bar{U}_2^* = \min(U, c_\alpha, c_\beta) \quad (\text{A13})$$

where  $z(c) = (H(c)/h(c))$ ,  $c_\alpha$  and  $c_\beta$  satisfy

$$c_\alpha + z(c_\alpha) + s = p \quad (\text{A14})$$

$$(p - c_\beta - z(c_\beta) - s)q^*(c_\beta) - p \int_0^{q^*(c_\beta)} F(x) dx = \Pi_M^- + \Pi_R^- \quad (\text{A15})$$

**Proof of Corollary 2.** From Proposition 3(a), we take derivative of  $r_2^*(c)$  with respect to  $c$  and then insert  $\dot{w}_2(c) = g(c)\dot{r}_2(c)$  from Proposition 3(b), which gives us a first-order differential equation

$$\frac{dr_2^*(c)}{dc} + A(c)r_2^*(c) + B(c) = 0$$

where  $g(c)$ ,  $A(c)$ , and  $B(c)$  are all defined in the text. The solution to the above equation is

$$r_2^*(c) = \frac{\int_D^c e^{\int_D^{c'} A(c') dc'} B(c') dc' + \beta}{e^{\int_D^c A(c') dc'}}$$

which is Eq. (20) in the text. As a result, Eq. (21) provides the solution to  $w_2^*(c)$ . The only undetermined factor is  $\beta$ ; this is due to the fact that there is an arbitrary factor between the wholesale price and the buyback price for Problem 2 while their joint function,  $q^*(c) = Q(w_2^*(c), r_2^*(c) | c)$ , finally determines the result of the contract. We select  $\beta$  in such a way that the buyback price will not be negative in the feasible region of  $c$ .

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