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Strategic sourcing for the short-lifecycle products

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ABSTRACT

Motivated by the sourcing of integrated circuits in the electronics industry, we study sourcing strategies for short-lifecycle products with two substitutable parts. The first part, referred to as the fast part, is highly responsive while having negligible fixed cost but high variable cost. The second part, referred to as the slow part, is opposite in these properties. We build models starting with the fast part to target the initial market, then switching to the pre-ordered slow part for volume production, and eventually transitioning back to the fast part until the product's end of lifecycle. Assuming an optional second order for the slow part, we model the sourcing process by a two-stage stochastic program. The thresholds for the fixed costs and the optimal ordering policies for the two orders are exactly derived. Assuming the demand throughout the product lifecycle as a multivariate Normal distribution, we approximately compute the policy parameters and expected profit for the two-order problem. In comparison to the fast-part only strategy and one-order slow part strategy, the second order of the slow part could be of great value if the demand correlation across time is high and/or the cost difference between the two parts is large. We also study the joint impact of fixed cost and leadtime as well as demand variation on the sourcing strategies.

1. Introduction

The continuous innovation in high-technology industries has made product lifecycles shorter and shorter, with a typical lifecycle lasting only 9 to 15 months. In such a fast-evolving market, part management is critical both from the supply chain perspective and from the product-development perspective. To accelerate a product's time-to-market, manufacturers have long used rapid prototyping, which employs a responsive yet flexible technology to iteratively debug the design and manufacturing process, enabling the product to be launched more quickly. This more flexible technology is often more expensive than an inflexible technology that, used alone, would take longer to debug and bring to market. Integrated circuits (ICs) are the prototypical example of a part that benefits from rapid prototyping. In the case of integrated circuits, a more flexible part is the field programmable gate array (FPGA), while the application-specific integrated circuit (ASIC) is the less expensive but less flexible part.

We briefly review the ASIC and FPGA technologies according to the pedagogical book by Smith (1997). ASICs are hardware-based and need a fab to complete the production. For ASICs, there are specific design and manufacturing requirements for a component called the "mask", which is tailored to each customer's order. A mask has a finite lifetime and is often not reusable. It must be prepared at high expense whatever the customer order size is, and so ASIC vendors usually require the buyer pay the upfront, non-recurring engineering costs (NRE), including the mask cost, software tooling, design verification, and prototype sample. The leadtime for an ASIC from design to production is typically 2–5 months, and the manufacturing cost is determined by the fabrication process.

FPGA companies are software-based and typically "fabless." Namely, chip manufacturers make the standard hardware (chips), for which the FPGA companies often build sufficient stocks. Once the FPGA developers receive orders, they only need to develop (program) the FPGA structures on the hardware, which is immediate to complete, enabling fast stocking of FPGAs in the sales channels and rapid penetration of the customer base (Trenz Electronic, 2001). The chip manufacturer's fixed cost, much lower than that for ASICs, is often amortized over a large number of customers and chips, so the fixed cost per chip is minimal. On the other hand, its manufacturing cost is usually much higher than an ASIC due to the lower density and lower yield rate. With the negligible fixed cost and fast responsiveness, an FPGA supplier is very flexible.

According to the EE Times, 90% of ASICs in production employed FPGAs at an earlier point in the development cycle (Jaeger, 2007). There are also practices in which electronics companies prototype the products with FPGAs, start volume

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production with ASICs, and then transition back to FPGAs to supply the end-of-life products (Trenz Electronic, 2001; Altera Company, 2009). The trends of FPGA and ASIC technologies are converging in that FPGAs have been changing their role from prototyping to full production. In fact, FPGAs have been used throughout entire projects for some mainstream products arising from networking, telecommunication, and consumer electronics.

Motivated by the ASIC/FPGA transitions in industrial practice, we propose a sourcing strategy for a short-lifecycle product with two substitutable parts, a fast part which has high variable cost but negligible fixed cost and leadtime and a slow part which is opposite in these three attributes. In the most general case, the part supplying process is conducted in three steps: (1) we begin with a fast part to catch time-to-market; (2) as soon as possible, we switch to a slow part which is cheap in unit cost for economical volume production; (3) finally, we allow a switch back to the fast part to mitigate demand uncertainty at the end of product lifecycle. To reflect the larger order sizes for the slow part and the short product lifecycle, we will restrict the slow part to be ordered at most twice. The goal of this paper is to find the optimal sourcing strategy with two operationally opposite suppliers (parts) throughout the product lifecycle. In the ASIC/FPGA case, the fixed costs of the two orders of the slow part differ significantly as the first order requires high upfront payment for design of the mask. However, we will relax this requirement and allow the two orders to exhibit any cost relationship. For instance, the second order could require the slow supplier to re-open the production facility after ceasing the production line, which may cost the OEM some extra money. We further assume that the variable cost remains unchanged for the two orders because the variable cost is mainly composed of labor cost and material cost which should not change much during a short period.

The sourcing strategies proposed in this paper may be adopted not only at the component level but also at the plant level. Lee and Hoyt (2001) present Lucent Technology's global supply chain evolution, in which a home plant is built when the market is immature while a plant in a developing country is built when the market becomes mature. Pisano and Rossi (1993) describe Ili Lilly's manufacturing plan where flexible plants are built at an earlier time to cover the initial demand for multiple products while specific plants are built at later times when the markets of the products become mature. Like our work, these two problems strive to integrate supplier management with product lifecycle management.

The paper is organized as follows. In Section 2, we introduce the relevant literature. In Section 3, we model the sourcing strategy with two possible orders from the slow supplier when both the fast and slow options are available. The optimal solution to the above model is characterized in Section 4. In Section 5, we present a stochastic programming approach to the two-order problem when the demand follows a multivariate Normal distribution. In Section 6, we conduct a number of numerical examples to illustrate the value of the second order and the impact of key parameters in the problem. Section 7 concludes the paper and discusses some further issues.

2. Literature review

There is little research that integrates product lifecycle management with supplier management though it has well been attempted in the industry. A recent paper sharing close motivation with us is Ulku et al. (2005), which studies the entry timing to an uncertain market for a short lifecycle product in the presence of make/buy options. There are two other related streams of research: inventory planning for short lifecycle product with one replenishment opportunity, and sourcing with two types of suppliers.

Much of the literature has treated the inventory problem for short-lifecycle or fashion products as a single-period newsvendor problem. However, a series of papers, pioneered by Murray and Silver (1966), have demonstrated through two-period models that a second order may improve profit dramatically if the demand forecast is updated after the first order is received (Bradford and Sugrue, 1990; Fisher and Raman, 1996; Eppen and Iyer, 1997). Moreover, Yan et al. (2003) consider a dual-supplier problem in the discrete time finite-horizon setting with forecast updating, and then they explicitly solve the two-period case. All of the aforementioned papers have studied the two-order newsvendor problem in which demand materializes only at the end of the planning duration and there is no fixed cost associated with each order. Fisher et al. (2001) propose a heuristic method to solve the two-order problem with demand updating in a continuous, finite-horizon model, which is applied to retail fashion. To enhance the solution of Fisher et al. (2001), Milner and Kouvelis (2002) frame the problem in a continuous-time model with a finite leadtime of the second order, assuming that the demand evolves as geometric Brownian motion and then proposing some approximate policies. On the other hand, Serel (2012) extends the two-order newsvendor problem to incorporate budget constraint.

The basic idea of dual sourcing is to hedge demand uncertainty with a portfolio of suppliers. The earliest papers, including Barankin (1961), Daniel (1962), and Neuts (1964), consider a periodic review system in which the regular supplier's leadtime is one period and the expedited supplier is instantaneous. Whittermore and Saunders (1977) derive the optimal policy for the situation that the regular and expedited leadtimes are multiples of the review period. A number of approximate solutions to this problem in a general setting has been proposed (Klosterhalfen et al., 2011, and the references therein). Bradley (2004) studies a production-inventory system in which a manufacturer subcontracts the production in case its own capacity is insufficient to satisfy demand. Allon and Van Mieghem (2010) design a dual-sourcing strategy for global operations, in which an off-shore supplier covers the average rate of demand while the near-shore supplier handles when demand surges. Bhatnagar et al. (2011) address the problem of coordinating aggregate planning and short-term scheduling in supply chains with both long-leadtime and short-leadtime supply modes where the fast mode can take advantage of demand forecast updating. The dual-sourcing literature has focused on infinite or long-horizon inventory systems, where demand is relatively stationary. Our paper studies dual sourcing for short-lifecycle products, for which demand could be highly non-stationary and managing the beginning and the end of the product lifecycle is critical.

The extreme scenario of dual sourcing, where the fast supplier is interpreted as the spot market, has been studied in the context of supplier/contract portfolio in the one-period (Seifert et al., 2004; Fu et al., 2010), two period (Peleg et al., 2002), and multi-period (Martinez-de-Albeniz and Simchi-Levi, 2005) settings. Such a procurement process was practiced at Hewlett–Packard, and Billington et al. (2002) interprets it in the frame of real options. Our paper also studies a portfolio of suppliers with significantly different operational properties; yet we explicitly embed the supplier selection and sourcing process in a demand profile which represents a short lifecycle product.

3. Framing the sourcing strategy

Assuming there are at most two orders of the slow part, we first state our assumptions and then depict the timing of events for sourcing a short-lifecycle product.

3.1. Assumptions and notations

Assumption 1. *For the slow part, (1) the second order is placed only if the first order is placed and only after the first order has arrived. (2) The second order incurs the same per-unit variable cost but may have different fixed cost than the first order.*

For narrative ease, from now on whenever we refer to the first or second orders without qualification, we are referring to the order of the slow part (supplier). The first part of Assumption 1 is quite natural in reality and it also avoids the order-crossing difficulty in analysis. The rationale for the second part of Assumption 1 has been analyzed in the introduction. For convenience, we assume that the second-order fixed cost is invariant to its timing.

Assumption 2. *(1) No fixed cost with the fast part; (2) the leadtime of the fast supplier is negligible.*

Given the zero fixed cost and fast responsiveness, we assume that the fast part could be ordered in the ongoing format, which is possible if a Just-in-Time (JIT) procurement process could be set up between the original equipment manufacturer (OEM) and the fast supplier. With such a part, it is possible to fulfill all the demands throughout the planning horizon as long as the product price is above the part cost. This is the key to the "*guaranteed availability*" for a real option mechanism in procurement postulated by Billington et al. (2002).

With Assumptions 1 and 2, we can explain the timing of events in the two-order case, followed by the fulfillment process throughout the product lifecycle, in Fig. 1.

In Fig. 1, we first discretize the entire planning horizon, from the beginning to the end of the product lifecycle. A period i is a duration $[i-1, i]$, starting with a point $i-1$ and ending with a point i along the timeline in Fig. 1. In the most general case, there are two orders of the slow part, which have leadtimes of L_1 and L_2, respectively. The first order is made at the beginning of the product lifecycle, and it arrives at the end of period 1. We employ a rolling horizon procedure for the second order. Namely, the second order is assumed to be placed at the beginning of period n ($n>1$), which is a decision variable, and it arrives at the end of period $n+L_2$; however, the real execution of this order might be altered after new demand information materializes. The end of the product lifecycle is labeled by period N. The demand arrives at the end of a period and then is fulfilled by the slow part if it is available or the fast part otherwise. Demand prior to period 1 is fulfilled by the fast part, which is to meet the time-to-market.

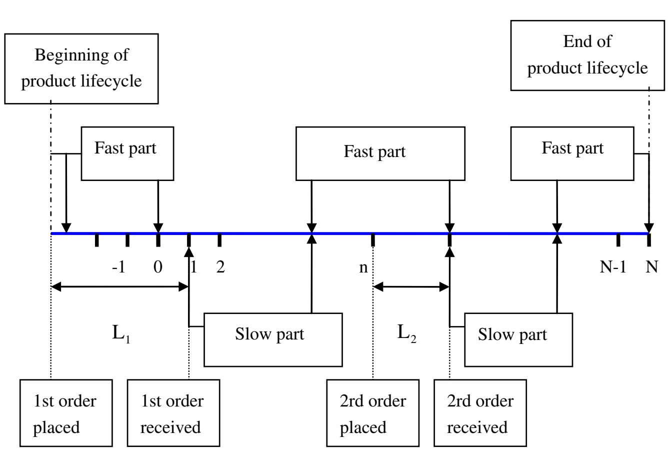

Fig. 1. Lifecycle sourcing with two substitutable parts.

It is therefore trivial for this duration in our problem and thus is not discussed further. Because the demand is uncertain, we are unable to predict when the slow part from the first order will be depleted. If the slow part is depleted before the second order arrives, we again adopt the fast part to supply the product. From the time that the second order arrives to the end of the product lifecycle, we repeatedly use the slow part first and then the fast part to supply the product if the former is depleted.

We use D_i to denote the forecasted demand in period i and define $f(D_i...D_j)$ as the joint demand distribution for periods i to j. The mean demand for period k in the interval $[i, j]$ is calculated as $\mu_k = \int...\int f(D_i...D_j) D_k dD_i...dD_j = \int D_k f(D_k) dD_k$. We further define $Y_{ij} = \sum_{k=i}^{j} D_k$ as the demand sum in period i to j and

$$f_{Y_{ij}}(x) = \int \cdots \int f(D_i \cdots D_j)\delta\left(x - \sum_{k=i}^{j} D_k\right) dD_i \cdots dD_j$$

as the corresponding probabilistic distribution, where $\delta(\cdot)$ is the impulse function satisfying $\int_{-\infty}^{\infty} \delta(x)dx = 1$, $\delta(0)=\infty$ and $\delta(x)=0$ if $x \neq 0$. The cumulative distribution function for Y_{ij} is defined as $F_{Y_{ij}}(x) = \int_0^x f_{Y_{ij}}(x')dx'$.

We define the remaining notations as:

k_1	fixed cost of the first order of the slow part,
c_s	unit variable cost of the slow part,
v	unit salvage value of the slow part,
h	unit holding cost of the slow part in a period, same for all the periods,
n	the period to place the second order,
k_2	fixed cost of the second order of the slow part, independent of the order period n,
c_f	unit variable cost of the fast part,
p_i	net selling price of the final product in period i with the cost of all other parts excluded,
Q_1,Q_2	first and second order quantities of the slow part, decision variables.

With regard to cost and price, we have the following assumption.

Assumption 3. (*1*) *The prices and costs in the problem satisfy* $p_1 \geq p_2 ... \geq p_N > c_f > c_s > v$.

3.2. Model

With the model setup stated earlier, we can define three sourcing strategies, namely, fast-part only strategy, one-order strategy and two-order strategy. For the fast-part only strategy, we supply a product by the fast part throughout its lifecycle, and therefore the expected profit is written as $\Pi_0 = \sum_{i=1}^{N}(p_i - c_f)\mu_i$. We have witnessed the fast-part only strategy used by telecommunication firms which adopt FPGA to support their products in short-term projects.

The second order is available only if there is the first order. We model the two-order strategy assuming the first order is present and the timing of the second order is fixed to occur in period n:

Problem P

$$\begin{aligned}\mathrm{Max}_{Q_1>0}\Pi_2(Q_1;n) = {} & -k_1 - c_s Q_1 + E_{D_1 \cdots D_{n+L_2-1}} \\ & \sum_{i=1}^{n+L_2-1}\Big[p_i D_i - c_f\Big(D_i - \min\Big(D_i, \Big(Q_1 - \sum_{j=1}^{i-1} D_j\Big)^+\Big)\Big) \\ & - h\Big(Q_1 - \sum_{j=1}^{i} D_j\Big)^+\Big] + E_{D_1 \cdots D_{n-1}} H_2(Q_1; D_1 \cdots D_{n-1}), \qquad (1)\end{aligned}$$

$$H_2(Q_1;D_1\cdots D_{n-1}) = \mathrm{Max}_{Q_2\geq 0}E_{D_n\cdots D_N|D_1\cdots D_{n-1}}\Big\{-k_2\theta(Q_2) -c_sQ_2+\sum_{k=n+L_2}^{N}[p_kD_k-c_f(D_k -\min(D_k,\Big(Q_2+\Big(Q_1-\sum_{j=1}^{n+L_2-1}D_j\Big)^+-\sum_{j=n+L_2}^{k-1}D_j)^+)\Big) -h(Q_2+\Big(Q_1-\sum_{j=1}^{n+L_2-1}D_j\Big)^+-\sum_{j=n+L_2}^{k}D_j)^+] +(h+v)\Big(Q_2+\Big(Q_1-\sum_{j=1}^{n+L_2-1}D_j\Big)^+-\sum_{j=n+L_2}^{N}D_j\Big)^+\Big\}. \quad (2)$$

in (2), $\theta(x)$ is the step function, which is equal to 1 if $x>0$, and 0 otherwise.

The objective in Problem P is to maximize the expected profit throughout the planning horizon, which is divided into two segments with the first segment including periods 1 to $n+L_2-1$ and the second segment including periods $n+L_2$ to N. In the first segment, there are fixed, variable and inventory costs associated with the first order of the slow part. In the part supplying process, we always first deplete the slow part and use the fast part only if the slow parts are exhausted. With the responsiveness of the fast part, we can fulfill all the demand and yield the expected revenue. The expected profit in the second segment is represented by (2), where the inventory of the slow part at the end of period $n+L_2$ is lifted by the second order quantity, Q_2. The part supplying process in the second segment is the same as in the first segment; in addition, in period N any existing slow parts are salvaged.

In Problem P, setting $Q_2=0$ automatically leads to the one-order strategy, which will be solved separately and used as a reference to the two-order strategy.

4. Solving the model

Problem P is a two-stage dynamic program. In a backward way, we first solve the second-order problem and then the first-order problem. In practice, we implement the first order and leave more opportunities for the second order upon new demand information.

4.1. Solving the second-order problem

The second-order problem in (2) finds the optimal quantity from the slow supplier for the demands from period $n+L_2$ to the end of the lifecycle, given the first-order quantity from the slow supplier and the demand realization in the first $n-1$ periods and the forecasted demand in periods n to $n+L_2$.

To solve (2), we first denote $I_{n-1}=I_{n-1}(Q_1;D_1\cdots D_{n-1})=(Q_1-\sum_{j=1}^{n-1}D_j)^+$, which also implies $(Q_1-\sum_{j=1}^{n+L_2-1}D_j)^+=(I_{n-1}-\sum_{j=n}^{n+L_2-1}D_j)^+$. Utilizing this property and

$$\min(D_k,(Q_2+(I_{n-1}-\sum_{j=n}^{n+L_2-1}D_j)^+-\sum_{j=n+L_2}^{k-1}D_j)^+) =[Q_2+(I_{n-1}-\sum_{j=n}^{n+L_2-1}D_j)^+-\sum_{j=n+L_2}^{k-1}D_j]^+ -[Q_2+(I_{n-1}-\sum_{j=n}^{n+L_2-1}D_j)^+-\sum_{j=n+L_2}^{k}D_j]^+,$$

we can simplify the second line of (2) and re-state (2) as

$$H_2(Q_1;D_1\cdots D_{n-1}) = \sum_{k=n+L_2}^{N}(p_k-c_f)\mu_k +c_fE_{D_n\cdots D_N|D_1\cdots D_{n-1}}\Big(I_{n-1}-\sum_{j=n}^{n+L_2-1}D_j\Big)^+ +\mathrm{Max}_{Q_2\geq 0}[-k_2\theta(Q_2)+G(Q_2;D_1\cdots D_{n-1})], \quad (3)$$

$$G(Q_2;D_1\cdots D_{n-1})=(c_f-c_s)Q_2-hE_{D_n\cdots D_N|D_1\cdots D_{n-1}}\sum_{k=n+L_2}^{N-1}\Big[Q_2+\Big(I_{n-1}-\sum_{j=n}^{n+L_2-1}D_j\Big)^+-\sum_{j=n+L_2}^{k}D_j\Big]^+ -(c_f-v)E_{D_n\cdots D_N|D_1\cdots D_{n-1}}[Q_2+\Big(I_{n-1}-\sum_{j=n}^{n+L_2-1}D_j\Big)^+ -\sum_{j=n+L_2}^{N}D_j]^+. \quad (4)$$

When we try to determine the optimal Q_2 in (3), we have two facts: (1) demands $(D_1...D_{n-1})$ are realized, and so is I_{n-1}; (2) demands for periods n and onwards are updated by the realized $(D_1...D_{n-1})$. In (4), $G(Q_2;D_1...D_{n-1})$ takes expectations on $(D_n...D_N)$ for which the joint distribution is parameterized by $(D_1...D_{n-1})$, and it also depends on I_{n-1}which is a function of $(D_1...D_{n-1})$. Therefore, the second-order problem formulated in (3) is in general path-dependent and non-Markovian.

Because of the non-Markovian property, there are infinite second-order problems even for a given state variable, I_{n-1}. For ease of presentation, we will frequently use $G(Q_2;I_{n-1})$ for $G(Q_2;D_1...D_{n-1})$ hereafter. We then have:

Proposition 1. *$G(Q_2;I_{n-1})$ is concave in Q_2 for given I_{n-1}. The optimal Q_2 to maximize $G(Q_2;I_{n-1})$ is given by the zero condition of the following:*

$$g(Q_2;I_{n-1})=(c_f-c_s)-h\sum_{k=n+L_2}^{N-1}F_{Y_{n,k}|D_1\cdots D_{n-1}}(Q_2) +\int_0^{I_{n-1}}f_{Y_{n,n+L_2-1}|D_1\cdots D_{n-1}}(x) \times[F_{Y_{n+L_2,k}|Y_{n,n+L_2-1}=x,D_1\cdots D_{n-1}}(Q_2+I_{n-1}-x) -F_{Y_{n+L_2,k}|Y_{n,n+L_2-1}=x,D_1\cdots D_{n-1}}(Q_2)]dx -(c_f-v)F_{Y_{n,k}|D_1\cdots D_{n-1}}(Q_2) +\int_0^{I_{n-1}}f_{Y_{n,n+L_2-1}|D_1\cdots D_{n-1}}(x) \times[F_{Y_{n+L_2,k}|Y_{n,n+L_2-1}=x,D_1\cdots D_{n-1}}(Q_2+I_{n-1}-x) -F_{Y_{n+L_2,k}|Y_{n,n+L_2-1}=x,D_1\cdots D_{n-1}}(Q_2)]dx. \quad (5)$$

Starting from the above proposition, the solution to the second order is stated in the following:

Theorem 1. *For a demand path $(D_1...D_{n-1})$, denoting*

$$S_2^n(D_1\cdots D_{n-1})=\mathrm{ArgMax}_{Q_2\geq 0}G(Q_2;I_{n-1}=0) \quad (6)$$

$$\Delta_2^n(D_1\cdots D_{n-1})=G(S_2^n(D_1\cdots D_{n-1});0), \quad (7)$$

the optimal solution to the second order is:

(1) *If $k_2\geq\Delta_2^n(D_1\cdots D_{n-1})$, then no second order;*
(2) *If $k_2<\Delta_2^n(D_1\cdots D_{n-1})$, there are reorder point, $s_2^n(D_1\cdots D_{n-1})$, and a reorder quantity, $q_2^n(D_1\cdots D_{n-1})$, such that*

$$q_2^n(D_1\cdots D_{n-1})=\mathrm{ArgMax}_{Q_2\geq 0}G(Q_2;s_2^n(D_1\cdots D_{n-1})) \quad (8)$$

$$G(q_2^n(D_1\cdots D_{n-1});\quad s_2^n(D_1\cdots D_{n-1}))=k_2+G(0;s_2^n(D_1\cdots D_{n-1})) \quad (9)$$

and (a) *If $I_{n-1}\geq s_2^n(D_1\cdots D_{n-1})$, then no second order,*
(b) *If $I_{n-1}<s_2^n(D_1\cdots D_{n-1})$, the order quantity is solved by*

$$Q_2(I_{n-1})=\mathrm{ArgMax}_{Q_2\geq 0}G(Q_2;I_{n-1}). \quad (10)$$

Theorem 1 partially resembles an (s,S) inventory policy, but its structure is more complicated. Part (1) says if the fixed cost is above a path-dependent threshold value, $\Delta_2^n(D_1\cdots D_{n-1})$, then we should not make the second order; this result is invariant to the inventory on hand. To be specific, we first determine $S_2^n(D_1 \cdots D_{n-1})$ by (6) as a path-dependent, order-up-to level with the starting inventory as zero, which determines $\Delta_2^n(D_1 \cdots D_{n-1})$ through (7). For part (2), if the fixed cost is below $\Delta_2^n(D_1 \cdots D_{n-1})$, then the on-hand inventory determines the second order. Namely, if the on-hand inventory is above another path-dependent threshold value, $s_2^n(D_1 \cdots D_n)$, then there should not be a second order; otherwise, a second order should be ordered in the quantity of (10). When there is no ambiguity, we will interchangeably use Q_2 and $Q_2(I_{n-1})$ in places to represent the second order which is a function of $I_{n-1}$and depends upon $(D_1 ... D_{n-1})$. The path dependence of the policy parameters obviously creates the curse-of-dimensionality problem, which will be approximately tackled to calculate the first order quantity.

As another observation, if the second order is non-zero, we are able to show:

Proposition 2. $-1 < \partial Q_2/\partial Q_1 \leq 0$.

The above proposition indicates that the second order will decrease by a mild degree when the first order increases. This also implies that the entire problem is more sensitive to the first order than to the second one, which will be further supported by the numerical examples in the computational section.

4.2. Solving the first-order problem

With Theorem 1 for the solution to the second order, we are able to present the first order problem by

Problem PP

$$\text{Max}_{Q_1>0} \Pi_2(Q_1,n) = -k_1 + \sum_{i=1}^{N}(p_i - c_f)\mu_i + H_1(Q_1),$$

$$\begin{aligned} H_1(Q_1) = {} & (c_f - c_s)Q_1 - h\sum\nolimits_{i=1}^{n+L_2-1} E_{D_1 \cdots D_{n+L_2-1}}\left(Q_1 - \sum\nolimits_{j=1}^{i} D_j\right)^+ \\ & + E_{D_1 \cdots D_{n-1}} E_{D_n \cdots D_N | D_1 \cdots D_{n-1}}[1_{Q_2(I_{n-1})>0}(-k_2 + G(Q_2(I_{n-1}); I_{n-1})) \\ & + 1_{Q_2(I_{n-1})=0} G(0; I_{n-1})]. \end{aligned} \quad (11)$$

In the above, $1_x = 1$ if x is true; otherwise $1_x = 0$. In the second line of (11), we should distinguish three scenarios:

(1) if $I_{n-1} = 0$, then $Q_2(I_{n-1}) = S_2^n(D_1 \cdots D_{n-1})$;
(2) if $I_{n-1} \geq s_2^n(D_1 \cdots D_{n-1})$, then $Q_2(I_{n-1}) = 0$;
(3) if $0 < I_{n-1} < s_2^n(D_1 \cdots D_{n-1})$, then $Q_2(I_{n-1})$ is solved via (10).

It is crucial to enumerate these three cases in computation. Solving Problem PP is now equivalent to finding the optimal Q_1 to maximize $H_1(Q_1)$.

Fisher et al. (2001) showed that a two-period newsvendor model is neither convex nor concave even without a fixed cost. Problem PP has extended their model in some dimensions and therefore it is no surprise that it is neither concave nor convex in general. A standard approach to solving Problem PP is line search, which should be more efficient if we could reduce the feasible region for Q_1.

Before characterizing the solution to the first order, we study the one-order strategy which is indeed a special (feasible) solution to Problem PP. For such a case, we have

Lemma 1. *If there is only one order of the slow part, we define*

$$H_3(Q_1) = (c_f - c_s)Q_1 - h\sum_{i=1}^{N-1}\int_0^{Q_1} F_{Y_{1i}}(x)dx - (c_f - v)\int_0^{Q_1} F_{Y_{1N}}(x)dx \quad (12)$$

(1) *$H_3(Q_1)$is concave and has a unique maximal point, S_1.*
(2) *If $c_f \to c_s$, then $S_1 \to 0$; therefore, we will use the fast part only.*
(3) *When $c_f > c_s$, the optimal sourcing strategy for a one-order policy is determined by an (S_1, Δ_1) rule, where $\Delta_1 = H_3(S_1)$; (a) if $k_1 > \Delta_1$, source the product by the fast part; (b)if $k_1 < \Delta_1$, order the slow part up to S_1 and then supply the product by the fast part if needed. In this case, the expected profit is $\Pi_1(S_1) = -k_1 + \sum_{i=1}^{N}(p_i - c_f)\mu_i + H_3(S_1)$.*

Lemma 1 reflects a strategy that has been applied in the integrated-circuit related industry widely. In fact, companies like Altera are using a hybrid approach that transitions from FPGA to ASIC and then back to FPGA. From this lemma we will focus on the situation that $c_f > c_s$ hereafter.

With the aid of Lemma 1, we state the solution to the first order in Problem P or PP as:

Theorem 2. *Suppose the timing of the second order is in period n. There is an extreme point for Q_1 that globally maximizes $H_1(Q_1)$. Denoting such a point as S_1^n and defining $\Delta_1^n = H_1(S_1^n)$, we have (1) $S_1^n \leq S_1$, (2) $\Delta_1^n \geq \Delta_1$,* and *(3) if $k_1 \geq \Delta_1^n$, then it is optimal to source the product by the fast part throughout the product lifecycle; otherwise, it is optimal to order the slow parts up to S_1^n.*

Theorem 2 characterizes the solution to the first order with a chance to place the second order. It provides a threshold for the fixed cost of the first order, above which we should source the product by the fast part only. Once S_1^n is determined, the expected profit of the two-order strategy is

$$-k_1 + \sum_{i=1}^{N}(p_i - c_f)\mu_i + H_1(S_1^n).$$

To compute S_1^n by line search, we have identified S_1 as an upper bound, which is easy to compute by maximizing (12). We further need to find an efficient lower bound to restrict the search region. One lower bound is found by solving the following myopic solution for the first order:

$$\begin{aligned} \text{Max}_{Q_1 \geq 0} \Pi_{myopic}(Q_1; n) = {} & -k_1 - c_s Q_1 + E_{D_1 \cdots D_{n+L_2-1}} \\ & \left\{ \sum\nolimits_{i=1}^{n+L_2-1}\left[p_i D_i - c_f\left(D_i - \min\left(D_i, \left(Q_1 - \sum\nolimits_{j=1}^{i-1} D_j\right)^+\right)\right)\right.\right. \\ & \left.\left. -h\left(Q_1 - \sum\nolimits_{j=1}^{i} D_j\right)^+\right] + (h+v)\left(Q_1 - \sum\nolimits_{j=1}^{n+L_2-1} D_j\right)^+ \right\} \end{aligned} \quad (13)$$

The above myopic formulation decides a first-order quantity such that the expected profit for periods 1 to $n + L_2 - 1$ is maximized, where the product is still supplied by the two parts and the leftover of the slow part in period $n + L_2$ is salvaged at value of v. Such a supplying process simply ignores the demand in period $n + L_2$ and onwards, and therefore the optimal solution must set a lower bound for S_1^n. The solution to (13) is given by the following quantity:

$$\begin{aligned} Q_{L,n} = \text{ArgMax}_{Q_1 \geq 0} & \left[(c_f - c_s)Q_1 - h\sum\nolimits_{i=1}^{n+L_2-2} E_{D_1 \cdots D_{n+L_2-1}}\right. \\ & \left.\left(Q_1 - \sum\nolimits_{j=1}^{i} D_j\right)^+ - (c_f - v)E_{D_1 \cdots D_{n+L_2-1}}\left(Q_1 - \sum\nolimits_{j=1}^{n+L_2-1} D_j\right)^+\right]. \end{aligned}$$

As a result, we will use $[Q_{L,n}, S_1]$ as the search space to compute S_1^n.

Theorems 1 and 2 have completely characterized the sourcing strategy with possibly two orders of the slow part. If the second order is in period n, the optimal sourcing strategy is a policy composed of five parameters, $(S_1^n, \Delta_1^n, \Delta_2^n, s_2^n, S_2^n)$, where Δ_2^n, s_2^n, S_2^n are dependent of $(D_1 ... D_{n-1})$. If the timing of the second order is a decision variable, we first solve Problem PP for a fixed n, resulting in the optimal order quantity $Q_1^*(n)$. Writing $\Pi_2(n) = \Pi_2(Q_1^*(n), n)$, we find the optimal n, defined as n^*, by solving

$$n^* = \text{ArgMax}_{2 \le n \le N-L_2} \Pi_2(n) \quad (14)$$

In (14), we naturally assume that the second order is placed between period 2 and period $N-L_2$.

Once we find n^*, we can determine the optimal Q_1 by a backtracking method. In the rolling schedule, there is further flexibility with the timing of the second order, which is beyond the scope of this paper.

5. Approximate algorithm for multivariate normal distributed demand

Due to the path dependence of the second-order problem, we further explore the two-order problem by the standard approach in stochastic programming (SP). We assume the demand distribution in the N periods follows a multivariate Normal distribution since this is representative enough for practical applications and yet more tractable than other distribution profiles.

5.1. Some properties of multivariate Normal distribution

For a product with a short lifecycle, the demand usually exhibits high correlation throughout the planning horizon. Hence, we have:

Assumption 4. *The demands in the N periods satisfy a multivariate Normal distribution:*

$$\begin{pmatrix} D_1 \\ \vdots \\ D_N \end{pmatrix} \sim N(\mu,\Sigma)$$

where $\mu = \begin{pmatrix} \mu_1 \\ \vdots \\ \mu_N \end{pmatrix}$ *and* $\Sigma = \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1N} \\ \vdots & & \vdots \\ \sigma_{N1} & \cdots & \sigma_{NN} \end{bmatrix}$ *are the demand mean vector and covariance matrix, respectively.*

Assuming a symmetric covariance matrix, we write $\sigma_{ii} = \sigma_i^2$ and $\sigma_{ij} = \sigma_{ji} = \rho_{ij}\sigma_i\sigma_j$. With a multivariate Normal distribution, the demand for the latter periods could be updated with the realization of the demand in the earlier periods by updating the mean value and the covariance matrix. We first write

$$\mu'_1 = \sum_{i=1}^{n-1} \mu_i, \quad \mu'_2 = \sum_{i=n}^{n+L_2-1} \mu_i, \quad \mu'_3 = \sum_{i=n+L_2}^{k} \mu_i,$$
$$\mu'_4 = \sum_{i=1}^{n+L_2-1} \mu_i, \quad (15)$$

where $\mu'_1,\mu'_2,\mu'_3,\mu'_4$ are the means of $Y_{1,n-1}, Y_{n,n+L_2-1}, Y_{n+L_2,k}, Y_{1,n+L_2-1}$, respectively. We then write

$$\sigma'_{11} = \sum_{i=1}^{n-1}\sum_{j=1}^{n-1} \sigma_{ij} \quad \sigma'_{22} = \sum_{i=n}^{n+L_2-1}\sum_{j=n}^{n+L_2-1} \sigma_{ij}$$

$$\sigma'_{33} = \sum_{i=n+L_2}^{k}\sum_{j=n+L_2}^{k} \sigma_{ij} \quad \sigma'_{44} = \sum_{i=1}^{n+L_2-1}\sum_{i=1}^{n+L_2-1} \sigma_{ij}$$

$$\sigma'_{12} = \sigma'_{21} = \sum_{i=1}^{n-1}\sum_{j=n}^{n+L_2-1} \sigma_{ij} \quad \sigma'_{34} = \sigma'_{43} = \sum_{i=n+L_2}^{k}\sum_{j=1}^{n+L_2-1} \sigma_{ij} \quad (16)$$

where $\sigma'_{11},\sigma'_{22},\sigma'_{33},\sigma'_{44}$ are the variances of $Y_{1,n-1}, Y_{n,n+L_2-1}, Y_{n+L_2,k}, Y_{1,n+L_2-1}$, respectively, while σ'_{12} is the covariance between $Y_{1,n-1}$ and $Y_{n,n+L_2-1}$ and σ'_{34} is the covariance between $Y_{n+L_2,k}$ and $Y_{1,n+L_2-1}$.

According to Paolella (2007), the sum of any two subsets of $(D_1...D_N)$ with no common element constitutes a bi-normal distribution. Therefore,

$$Y_{n,n+L_2-1} | Y_{1,n-1} = y \sim N\left(\mu'_2 + \frac{\sigma'_{12}}{\sigma'_{11}}(y-\mu'_1), \sigma'_{22} - \frac{\sigma'_{12}\sigma'_{21}}{\sigma'_{11}}\right) \quad (17)$$

$$Y_{n+L_2,k} | Y_{1,n+L_2-1} = y \sim N\left(\mu'_3 + \frac{\sigma'_{34}}{\sigma'_{44}}(y-\mu'_4), \quad \sigma'_{33} - \frac{\sigma'_{34}\sigma'_{43}}{\sigma'_{44}}\right). \quad (18)$$

In (17), $Y_{n,n+L_2-1}$ is updated when $D_1...D_{n-1}$, or $Y_{1,n-1}$, is realized. In (18), $Y_{n+L_2,k}$ is updated when the demands in the durations of $[1,n-1]$ and $[n,n+L_2-1]$ are both known, or, equivalently, $Y_{1,n+L_2-1}$ is known. Finally, (17) and (18) are used to solve the zero condition of (5) numerically.

5.2. An approximate algorithm for the two-order problem

A line search is theoretically efficient to find the optimal Q_1. Nevertheless, there are two barriers to reaching an exact solution toward this: (1) we need to sum over an infinite number of paths for $(D_1...D_{n-1})$, though only a single random number, $Y_{1,n-1}$, is eventually needed in the solution; and (2) the solution to the second-stage problem, for a specific path of $\{D_1...D_{n-1}\}$, may take a long time, which will significantly constrain the solution capability. For these reasons, we have to choose the standard approximate approach in stochastic programming, i.e., to sample the demand profile. We present the high-level pseudo-code for such a stochastic program in Algorithm 1.

Algorithm 1: Approximate solutions to Q_1 and n.

0. *Input parameters; let Q_1, n be decision variables.*
1. *Sample the random space of $Y_{1,n-1}$ by truncating the demand in the region of $[L, D]$ and dividing it into S segments with the corresponding weight $f_{Y_{1,n-1}}(L+i(D-L/S)) \times (D-L/S)$ for $i=0,1\ldots,S-1$.*
2. For each sample of $Y_{1,n-1}$, or for each $i=0,1\ldots,S-1$, update $Y_{n,n+L_2-1}, Y_{n+L_2,k}$, $\forall k > n+L_2$ *according to (17), (18);*
   Solve the second-stage problem given Q_1 and n, with the solutions given by Theorem 1.
3. Based on the solutions to the second order for the sampled $Y_{1,n-1}$, derive the approximate, *expected profit for the periods from $n+L_2$ to N;*
   For a given n, derive the approximate profit function $H_1(Q_1)$ according to (11);
   For a given n, perform a line search for optimal Q_1 over $[Q_{L,n},S_1]$ to maximize $H_1(Q_1)$.
4. *Perform a line search over n to maximize $H_1(Q_1)$; return the optimal Q_1 and n.*

6. Numerical experiments

In this section, we try to gain further insights into the sourcing strategies through a set of numerical experiments assuming a multivariate Normal demand and a static timing of the second order.

Throughout this section, the period number N is set as 6, by which we imitate the typical four phases of a new product, namely (1) the product-launch phase, (2) the demand-ramp phase, (3) the peak-demand phase, and (4) the end-of-life phase. The prices of the final product are assumed to be linearly decreasing with p_1=\$10 and p_6=\$7; the unit cost of the slow part is set as c_s=\$2; the inventory cost with the slow part is h=\$0.2 per period, and the salvage value is v=\$0.2.We assume that the demand has a multivariate Normal distribution with $(\mu_1...\mu_6)$=(20, 30, 50, 30, 20, 10)and $\sigma_{ij} = \rho^{|i-j|}\sigma_i\sigma_j$, where $(\sigma_1...\sigma_6)$=(16, 24, 40, 24, 16, 8). With such a covariance matrix, the demand correlation decays with the interval between two periods. In addition, we will restrict to the cases of $\rho > 0$; namely, the demands are positively correlated across different periods. The case of $\rho<0$ can be extended with minor modification, and we will see that the results will be less interesting for the two-order strategy. We will first vary k_1, k_2, L_2, ρ, c_f, from which we will identify the key drivers that determine the optimal sourcing strategies. We will finally identify the precision of the approximation algorithm by varying ($\sigma_1 \ldots \sigma_6$) and ρ which the demand uncertainties stem from.

The computation is extensive, so we have to sacrifice the accuracy. In this example, we first choose $S=10$, which will be verified to be robust enough later on. We will also choose the error tolerance as one unit for the order quantity, which is about 1% for the accuracy requirement.

6.1. Value of the second order

In this numerical study, we first derive S_1 and $\Pi_1(S_1)$ from Lemma 1 for the one-order strategy, and then apply Algorithm 1 to solve the two-order solution with the optimal timing, order quantity of the first order, and expected profit, denoted as n,S_1^n, and $\Pi_2(S_1^n;n)$, respectively. We start with the case from the motivating problem where the first-order fixed cost is much higher than the second order's fixed cost. To determine the impact of the different parameters, we start with relatively low values of the fixed costs, $k_1=\$50$, $k_2=\$10$, and second-order leadtime, $L_2=1$. We choose the correlation factor ρ as 0.1, 0.5, and 0.9 while $c_f=\$4$ and \$6 throughout the numerical examples. We define $[\Pi_2(S_1^n;n)-\Pi_1(S_1)]/\Pi_1(S_1)$ in the percentage as the value of the two-order sourcing strategy against the one-order sourcing strategy. The outputs are depicted in Fig. 2.

In Fig. 2 we can see that the high c_f value lowers the profit of any sourcing approach in different magnitudes. The fast-part only approach is very sensitive to c_f but is unaffected by ρ because it depends only on the mean demands. The one-order strategy builds up inventory of the slow part with the safety stock for the demand uncertainty. The high ρ leads to high cumulative demand variation which is combated by high safety stock of the slow part or more supply of the fast part; therefore, the profit is decreasing in ρ in the one-order sourcing strategy. However, the one-order strategy does not need as many units of the fast part as the fast-part only strategy, so the expected profit is decreasing in c_f but not as significant as in the fast-part only strategy.

For the two-order strategy, its profit is only slightly sensitive to c_f because the second order has largely combated the upside of demand variation, and therefore very few units of the fast part are used on average. In addition, the higher ρ, the more valuable is the two-order strategy against the one-order strategy. The reason is that the second order is used only when the forecasted demand is high enough so it is more valuable when ρ is high. Finally, the two-order strategy is more valuable against the one-order strategy when c_f is high because the one-order strategy is likely to use more of the fast part and therefore its value deteriorates with increasing c_f.

6.2. Role of the fixed cost and leadtime

From Lemma 1 and Theorem 2, k_1 determines the borders between the fast-part only strategy and the strategies with the slow part. The second order is basically decided by k_2 and L_2 in addition to the demand realization. In this experiment, we investigate the effects of fixed costs and leadtime. We first preserve all the data in Fig. 1 in the case of $c_f=\$4$ but vary k_2 by \$10, \$35, and \$60, which represents that k_2 is 20%, 70%, and 120%, respectively, of k_1. The results are drawn in Fig. 3.

From Fig. 3, we find that the profit of the two-order strategy is not very sensitive to k_2, particularly when ρ is small; this is because the second order is made only for a small set of demand realizations. When ρ is larger, it is more likely to use the second order, so it is more sensitive to k_2. Even in this case the two-order strategy is not heavily sensitive to k_2 for the relatively low values of k_1 and k_2.

In addition to $k_1$and k_2, L_2 in Fig. 3 is also small. In Fig. 4, we complete another experiment with a higher $k_1$and high-low combinations of k_2 and L_2. In reference to Fig. 3, we find from

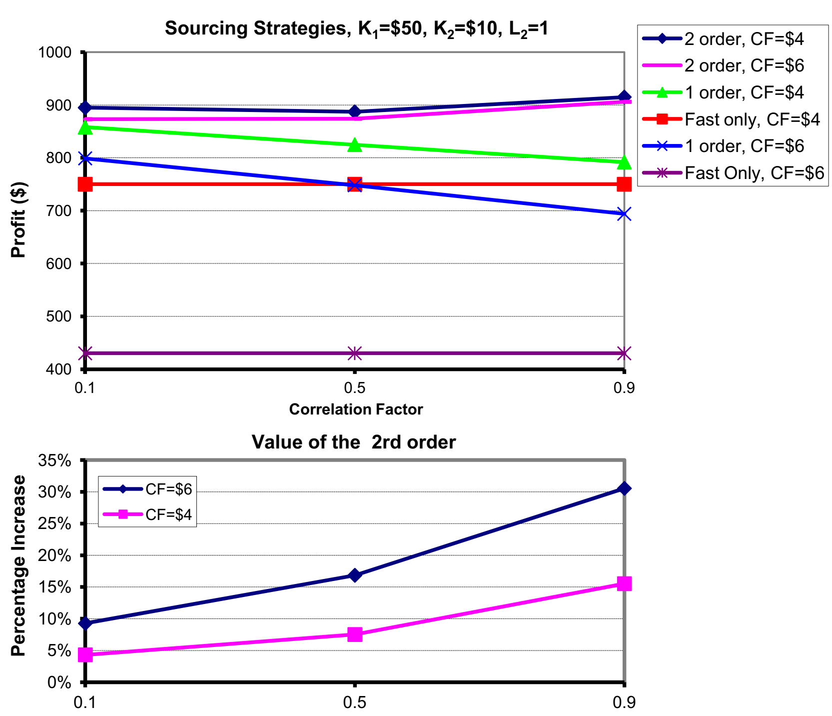

Fig. 2. Value of the second order.

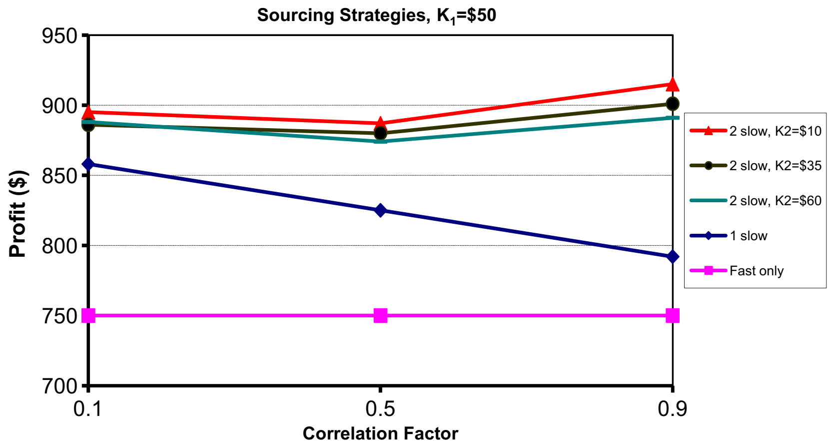

Fig. 3. The effect of second-order fixed cost.

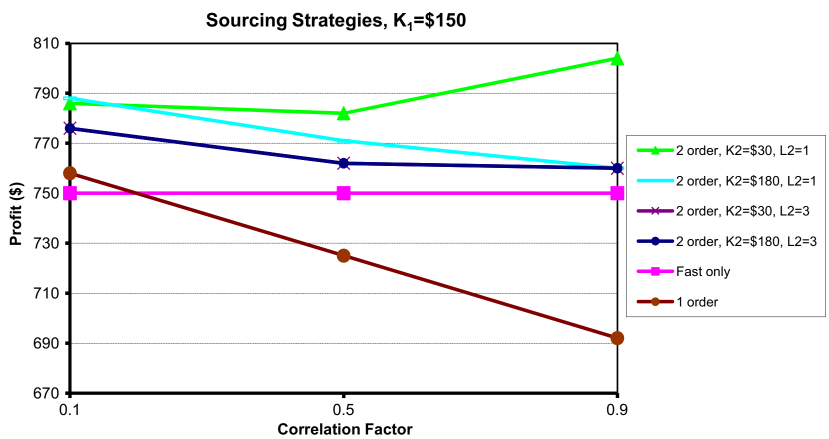

Fig. 4. The joint effect of the second-order fixed cost and leadtime.

Fig. 4 that the high k_1 can make the fast-part only strategy outperform the one-order strategy. Nevertheless, the two-order strategy still outperforms the fast-part only strategy for various values of k_2 and L_2. The two-order strategy is also insensitive to k_2 for high L_2 because in this case the second order is less likely made to cover the demand in limited periods.

From Figs. 2–4, we find a common property that when $\rho \to 0$ the one-order strategy and the two-order strategy have a converging trend, which means less chance to make the second order in this case. This trend is perceived more significant when ρ becomes negative, which is less interesting for further study.

6.3. Precision of stochastic programming

All the numerical results are based on the stochastic program in Algorithm 1 and therefore depend on the sampling process of the random variable $Y_{1,n-1}$, or the parameter S. Since the tested case in Fig. 2 with k_1=\$50, k_2=\$10, L_2=1 and c_f=\$6 delivers the most significant value of the second order, we test the robustness of stochastic programming based on this case by choosing S as 5, 10 and 15. The demand uncertainty in the problem is jointly determined by ($\sigma_1 ... \sigma_6$) and ρ, so the numerical study with regard to stochastic programming consists of two parts. In part 1, we preserve the demand variance as before but change ρ as 0.1, 0.5, 0.9. In part 2, we keep ρ=0.9 and change ($\sigma_1 ... \sigma_6$) by three cases: (I) (8, 12, 20, 12, 8, 4), (II) (12, 18, 30, 18, 12, 6), and (III) (16, 24, 40, 24, 16, 8). The results of the two parts are combined in Fig. 5.

From Fig. 5, we find the stochastic program in Algorithm 1 is robust enough with respect to the sampling size, S.

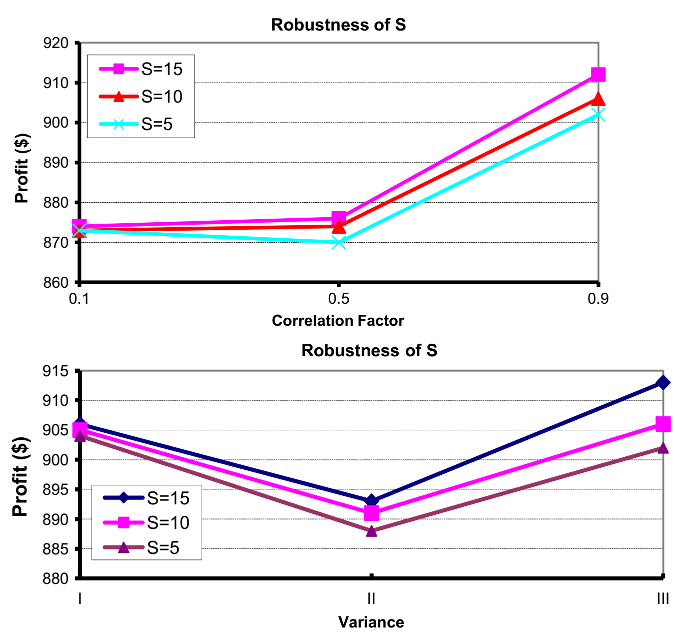

Fig. 5. The effect of approximation precision.

An interpretation is that the second order and the fast part have eliminated large portions of uncertainties. In a more precise manner, the two-order result is in general more affected by S when the demand correlation or the variance is high. The reason is that either of these two factors leads to high demand variations for the cumulative demands throughout the product lifecycle, when the role of sampling size is more important. However, even in this case, the expected profit only differs by less than 0.5% when S increases from 5 to 10 and from 10 to 15. Another minor observation is that when the demand correlation or the demand variance is neither too high nor too low the expected profit of the two-order strategy is relatively low. Actually, if the demand uncertainty is low the second order is of little use whereas it could be of great use when this uncertainty is high. However, in the intermediate case, it is uneconomical to use the second order; yet, it has moderate demand uncertainty which is not well tackled.

7. Discussion

Motivated by component sourcing in the electronics industry, we have modeled a general framework to source short-lifecycle products with a slow part and a fast part, assuming there are two potential orders of the slow part. Two special solutions to this general approach include the fast-part only strategy and the one-order strategy which both have been adopted in industry. A second order of the slow part could add significant value to the system when the demand correlation across time is high and/or the difference between the variable costs of the two parts is high, largely because we have efficiently applied the second order to the upside of the demand uncertainty. With the existence of the first order, we find the second order is more sensitive to the leadtime than to the fixed cost. The results in this paper indicate the potential value to apply the two-order strategy in practice.

In contrast to the planning for fashion products where final products are demanded at the end of the selling season, our sourcing strategy needs to cover the demand spread across the entire lifecycle of the product where no backorders are allowed. In our framework, the product lifecycle has been modeled by a generic demand profile. To build our model with some well-accepted lifecycle models should add more venues to this research. It is also of interest to apply the two-order sourcing strategy to business settings like electrical-components industry. When we implement the model using a rolling schedule, we have more flexible timing with the second order. In fact, if the optimal timing of the second order from (14) is to execute this order immediately, i.e., $n^{*}=2$, then this order is made. However, if the optimal timing is to make the order in a latter period, we have the chance to adjust this optimal timing after we receive the first order and after some demand is materialized. With such an implementation process, we expect the value of the second order should be more significant than pre-setting the timing of the second order.

The lifecycle sourcing framework in this paper has some assumptions that are approximations of reality. These include but are not limited to: (1) the pricing of the final products might be fluctuating in reality; (2) the variable costs of the slow parts of the two orders might be different; (3) the product lifecycle itself is uncertain; and (4) more than two orders of the slow part are allowed as long as the leadtime conditions permit such a choice. All of these factors may complicate the problem setting, for which we need to enhance the models in this paper. Our model also is an example to study the non-stationary inventory systems which were not studied sufficiently in the literature.

Finally, although we use fast and slow parts to represent the two suppliers in framing the sourcing strategy, we can interpret them as make/buy or off-shore/in-shore sourcing options. Therefore, to integrate the market conditions with such a pair of suppliers has broader meaning than the motivating problem from electronics industry.

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Appendix A

A.1. Proof of Proposition 1

Using

$$\frac{\partial[Q_2-a]^+}{\partial Q_2}=1_{Q_2>a},\quad \frac{\partial^2[Q_2-a]^+}{\partial Q_2^2}=\frac{\partial 1_{Q_2>a}}{\partial Q_2}=\delta(Q_2-a)$$

$$\frac{\partial G(Q_2;D_1\cdots D_{n-1})}{\partial Q_2}=(c_f-c_s)-hE_{D_n\cdots D_N|D_1\cdots D_{n-1}}\sum_{k=n+L_2}^{N-1}1_{Q_2+(I_{n-1}-\sum_{j=n}^{n+L_2-1}D_j)^+-\sum_{j=n+L_2}^{k}D_j>0}$$
$$-(c_f-v)E_{D_n\cdots D_N|D_1\cdots D_{n-1}}1_{Q_2+(I_{n-1}-\sum_{j=n}^{n+L_2-1}D_j)^+-\sum_{j=n+L_2}^{N}D_j>0}$$
$$\frac{\partial^2 G(Q_2;D_1\cdots D_{n-1})}{\partial Q_2^2}=-hE_{D_n\cdots D_N|D_1\ldots D_{n-1}}\sum_{k=n+L_2}^{N-1}\delta(Q_2+(I_{n-1}-\sum_{j=n}^{n+L_2-1}D_j)^+-\sum_{j=n+L_2}^{k}D_j)$$
$$-(c_f-v)E_{D_n\cdots D_N|D_1\cdots D_{n-1}}\delta(Q_2+(I_{n-1}-\sum_{j=n}^{n+L_2-1}D_j)^+-\sum_{j=n+L_2}^{N}D_j)<0.$$

hence, $G(Q_2;D_1\ldots D_{n-1})$is concave, and the first-order derivative yields the optimal solution. But for any $k\geq n+L_2$,

$$E_{D_n\cdots D_N|D_1\cdots D_{n-1}}1_{Q_2+(I_{n-1}-\sum_{j=n}^{n+L_2-1}D_j)^+-\sum_{j=n+L_2}^{k}D_j>0}$$
$$=E_{D_n\cdots D_N|D_1\cdots D_{n-1}}1_{Q_2+I_{n-1}-\sum_{j=n}^{n+L_2-1}D_j>0}1_{I_{n-1}-\sum_{j=n}^{n+L_2-1}D_j>0}$$
$$+E_{D_n\cdots D_N|D_1\cdots D_{n-1}}1_{Q_2-\sum_{j=n+L_2}^{k}D_j>0}1_{I_{n-1}-\sum_{j=n}^{n+L_2-1}D_j\leq 0}$$
$$=E_{D_n\cdots D_N|D_1\cdots D_{n-1}}1_{Q_2-\sum_{j=n+L_2}^{k}D_j>0}$$
$$+E_{D_n\cdots D_N|D_1\cdots D_{n-1}}(1_{Q_2+I_{n-1}-\sum_{j=n}^{k}D_j>0}-1_{Q_2-\sum_{j=n+L_2}^{k}D_j>0})1_{I_{n-1}>\sum_{j=n}^{n+L_2-1}D_j}$$
$$=E_{D_n\cdots D_N|D_1\cdots D_{n-1}}1_{Q_2-\sum_{j=n+L_2}^{k}D_j>0}$$
$$+E_{D_n\cdots D_N|D_1\cdots D_{n-1}}1_{I_{n-1}>\sum_{j=n}^{n+L_2-1}D_j,Q_2<\sum_{j=n+L_2}^{k}D_j<Q_2+I_{n-1}-\sum_{j=n}^{n+L_2-1}D_j}$$
$$=F_{Y_{n,N}|D_1\cdots D_{n-1}}(Q_2)+\int_0^{I_{n-1}}f_{Y_{n,n+L_2-1}|D_1\ldots D_{n-1}}(x)$$
$$[F_{Y_{n+L_2,k}|Y_{n,n+L_2-1}=x,D_1\cdots D_{n-1}}(Q_2+I_{n-1}-x)$$
$$-F_{Y_{n+L_2,k}|Y_{n,n+L_2-1}=x,D_1\cdots D_{n-1}}(Q_2)]dx.$$

From above, the optimal Q_2 is given by solving $g(Q_2;I_{n-1})=\partial G(Q_2;D_1\ldots D_{n-1})/\partial Q_2=0$, where

$$g(Q_2;I_{n-1})=(c_f-c_s)-h\sum_{k=n+L_2}^{N-1}F_{Y_{n,k}|D_1\cdots D_{n-1}}(Q_2)$$
$$+\int_0^{I_{n-1}}f_{Y_{n,n+L_2-1}|D_1\ldots D_{n-1}}(x)[F_{Y_{n+L_2,k}|Y_{n,n+L_2-1}=x,D_1\cdots D_{n-1}}$$
$$\times(Q_2+I_{n-1}-x)\quad -F_{Y_{n+L_2,k}|Y_{n,n+L_2-1}=x,D_1\cdots D_{n-1}}(Q_2)]dx$$
$$-(c_f-v)F_{Y_{n,N}|D_1\cdots D_{n-1}}(Q_2)$$
$$+\int_0^{I_{n-1}}f_{Y_{n,n+L_2-1}|D_1\cdots D_{n-1}}(x)[F_{Y_{n+L_2,N}|Y_{n,n+L_2-1}=x,D_1\cdots D_{n-1}}$$
$$\times(Q_2+I_{n-1}-x)\quad -F_{Y_{n+L_2,N}|Y_{n,n+L_2-1}=x,D_1\cdots D_{n-1}}(Q_2)]dx \tag{A1}$$

which is (5) in the text.

A.2. Proof of Theorem 1

Starting from (5), we characterize the second order by two Lemmas.

Lemma A1.

(1) If $I_{n-1}\to 0$, then $g(0;\ I_{n-1})>0$; and for any I_{n-1}, $g(\infty;I_{n-1})<0$;
(2) For any I_{n-1}, $\partial g(Q_2;I_{n-1})/\partial Q_2<0$; (3) For any Q_2, $g(Q_2;\infty)<0$ and $\partial g(Q_2;I_{n-1})/\partial I_{n-1}<0$.

Proof of Lemma A1

(1) From (A1), if $I_{n-1}\to 0$, then $g(0;I_{n-1})\to c_f-c_s>0$.
Moreover, for any I_{n-1}, if $Q_2=\infty$, then $E_{D_n\cdots D_N|D_1\cdots D_{n-1}}1_{Q_2+(I_{n-1}-\sum_{j=n}^{n+L_2-1}D_j)^+-\sum_{j=n+L_2}^{k}D_j>0}=1$, so

$$g(\infty;I_{n-1})=(c_f-c_s)-(N-n-L_2-1)h-(c_f-v)<0.$$

(2) $\partial g(Q_2;I_{n-1})/\partial Q_2<0$ is from the concavity of $G(Q_2;I_{n-1})$.
(3) For any Q_2, $g(Q_2;\infty)=(c_f-c_s)-(N-n-L_2-1)h-(c_f-v)<0$.
Moreover, we rewrite

$$g(Q_2;I_{n-1})=(c_f-c_s)-hE_{D_n\cdots D_N|D_1\cdots D_{n-1}}\sum_{k=n+L_2}^{N-1}1_{Q_2+(I_{n-1}-\sum_{j=n}^{n+L_2-1}D_j)^+-\sum_{j=n+L_2}^{k}D_j>0}-(c_f-v)E_{D_n\cdots D_N|D_1\cdots D_{n-1}}1_{Q_2+(I_{n-1}-\sum_{j=n}^{n+L_2-1}D_j)^+-\sum_{j=n+L_2}^{N}D_j>0}$$

For any $k\ge n+L_2$,

$$\frac{\partial}{\partial I_{n-1}}E_{D_n\cdots D_N|D_1\cdots D_{n-1}}1_{Q_2+(I_{n-1}-\sum_{j=n}^{n+L_2-1}D_j)^+-\sum_{j=n+L_2}^{k}D_j>0}=E_{D_n\cdots D_N|D_1\cdots D_{n-1}}\delta\Big(Q_2+(I_{n-1}-\sum_{j=n}^{n+L_2-1}D_j)^+-\sum_{j=n+L_2}^{k}D_j\Big)1_{I_{n-1}-\sum_{j=n}^{n+L_2-1}D_j>0}\ge 0$$

Therefore, $\partial g(Q_2;I_{n-1})/\partial I_{n-1}\le 0$ from the expression of $g(Q_2;I_{n-1})$.

Lemma A2. $\partial G(Q_2(I_{n-1});I_{n-1})/\partial I_{n-1}<0$.

Proof of Lemma A2. By definition, $Q_2(I_{n-1})$ solves $g(Q_2(I_{n-1});I_{n-1})=0$, so

$$\frac{\partial G(Q_2(I_{n-1});I_{n-1})}{\partial I_{n-1}}=g(Q_2(I_{n-1});I_{n-1})\frac{\partial Q_2(I_{n-1})}{\partial I_{n-1}}-hE_{D_n\cdots D_k|D_1\cdots D_{n-1}}\sum_{k=n+L_2}^{N-1}1_{Q_2+I_{n-1}-Y_{n,k}>0,I_{n-1}>Y_{n,n+L_2-1}}-(c_f-v)E_{D_n\cdots D_N|D_1\cdots D_{n-1}}1_{Q_2+I_{n-1}-Y_{n,N}>0,I_{n-1}>Y_{n,n+L_2-1}}$$
$$=-hE_{D_n\cdots D_k|D_1\cdots D_{n-1}}\sum_{k=n+L_2}^{N-1}1_{Q_2+I_{n-1}-Y_{n,k}>0,I_{n-1}>Y_{n,n+L_2-1}}-(c_f-v)E_{D_n\cdots D_N|D_1\cdots D_{n-1}}1_{Q_2+I_{n-1}-Y_{n,N}>0,I_{n-1}>Y_{n,n+L_2-1}}<0$$

So Lemma A2 is proved.

Lemmas A1 and A2 assist us in characterizing the solution to Q_2 from the zero condition of (5). For Lemma A1, part (1) infers that it has a positive, finite solution if $I_{n-1}\to 0$; from part (2), $g(Q_2;I_{n-1})$ decreases in Q_2, so if $g(0;I_{n-1})\le 0$ then there is no finite solution for Q_2 with a given I_{n-1}; part (3) implies that $g(0,I_{n-1})$ decreases in I_{n-1} and is negative if I_{n-1} goes to infinity. These results infers that there is a threshold value for I_{n-1}, if and only if below which there is a finite solution to solve (5) in the text. Denoting this threshold value as $s_2^n(D_1\cdots D_{n-1})$ which needs to be determined, the feasible region for $I_{n-1}$to have an order is $[0,\ s_2^n(D_1\cdots D_{n-1})]$.

We then solve $S_2^n(D_1\cdots D_{n-1})=\text{ArgMax}_{Q_2\ge 0}G(Q_2;I_{n-1}=0)$ and define $\Delta_2^n(D_1\cdots D_{n-1})=G(S_2^n(D_1\cdots D_{n-1});0)$. Since $G(Q_2(I_{n-1});I_{n-1})$decreases with respect to I_{n-1} from Lemma A2, if $k_2>\Delta_2^n(D_1\cdots D_{n-1})$ then for any I_{n-1}there needs no second order. Otherwise, we need to determine $s_2^n(D_1...D_{n-1})$, which is solved from the following pair of equations

$$q_2^n(D_1\cdots D_{n-1})=\text{ArgMax}_{Q_2\ge 0}G(Q_2;s_2^n(D_1\cdots D_{n-1}))\quad\text{(A2)}$$

$$G(q_2^n(D_1\cdots D_{n-1});\quad s_2^n(D_1\cdots D_{n-1}))=k_2+G(0;s_2^n(D_1\cdots D_{n-1}))\quad\text{(A3)}$$

In the above, (A2) is a repetition of the zero condition for (A1) with $I_{n-1}=s_2^n(D_1\cdots D_{n-1})$; (A3) says in this situation the profit increase of the second order will be deducted by the fixed cost.

With Lemma A2, For any $I_{n-1}\in[0,s_2^n(D_1\cdots D_{n-1})]$, the optimal solution is to make an order and the order quantity satisfies $Q_2(I_{n-1})=\text{ArgMax}_{Q_2\ge 0}G(Q_2;I_{n-1})$.

Theorem 1 is thus proved.

A.3. Proof of Proposition 2

Since $I_{n-1}=(Q_1-\sum_{j=1}^{n-1}D_j)^+$, we have $\frac{\partial I_{n-1}}{\partial Q_1}=1_{Q_1>\sum_{j=1}^{n-1}D_j}$.

Since Q_2 is a function of I_{n-1} and $g(Q_2(I_{n-1});I_{n-1})=0$ as stated in Lemma A2, we have $\partial g(Q_2(I_{n-1});I_{n-1})/\partial I_{n-1}=0$, or $(\partial g(Q_2;I_{n-1})/\partial Q_2)(\partial Q_2/\partial I_{n-1})+\partial g(Q_2;I_{n-1})/\partial I_{n-1}=0$, from which we have

$$\frac{\partial Q_2/\partial I_{n-1}}{\partial Q_2/\partial I_{n-1}+1}=-A/B<0,$$

where

$$A=h\sum_{k=n+L_2}^{N-1}\int_0^{I_{n-1}}f_{Y_{n,n+L_2-1}|D_1\ldots D_{n-1}}(x)f_{Y_{n+L_2,k}|Y_{n,n+L_2-1}=x,D_1\ldots D_{n-1}}(Q_2+I_{n-1}-x)dx+(c_f-v)\int_0^{I_{n-1}}f_{Y_{n,n+L_2-1}|D_1\ldots D_{n-1}}(x)f_{Y_{n+L_2,N}|Y_{n,n+L_2-1}=x,D_1\ldots D_{n-1}}(Q_2+I_{n-1}-x)dx$$
$$B=h\sum_{k=n+L_2}^{N-1}f_{Y_{n,k}|D_1\ldots D_{n-1}}(Q_2)+(c_f-v)f_{Y_{n+L_2,N}|Y_{n,n+L_2-1}=x,D_1\ldots D_{n-1}}(Q_2)$$

hence $-1<\partial Q_2/\partial I_{n-1}<0$. Also, $\partial Q_2/\partial Q_1=1_{Q_1>\sum_{j=1}^{n-1}D_j}(\partial Q_2/\partial I_{n-1})$, so $-1<\partial Q_2/\partial Q_1\le 0$.

A.4. Proof of Lemma 1

Inserting (4) into (11) while setting $Q_2=0$, we find $\Pi_2(Q_1,\ n)=-k_1+H_3(Q_1)$where

$$H_3(Q_1)=(c_f-c_s)Q_1-h\sum_{i=1}^{n+L_2-1}E_{D_1\cdots D_{n+L_2-1}}(Q_1-\sum_{j=1}^{i}D_j)^+-E_{D_1\cdots D_N}\left[h\sum_{i=n+L_2}^{N-1}\Big(Q_1-\sum_{j=1}^{i}D_j\Big)^++(c_f-v)\Big(Q_1-\sum_{j=1}^{N}D_j\Big)^+\right]$$
$$=(c_f-c_s)Q_1-h\sum_{i=1}^{N-1}\int_0^{Q_1}F_{Y_{1i}}(x)dx-(c_f-v)\int_0^{Q_1}F_{Y_{1N}}(x)dx$$

All the other results in this lemma are then easy to derive.

A.5. Proof of Theorem 2

(1) From Theorem 1, we have

$$H_1(Q_1) = (c_f - c_s)Q_1 - h\sum_{i=1}^{n+L_2-1} E_{D_1\cdots D_{n+L_2-1}}\left(Q_1 - \sum_{j=1}^{i} D_j\right)^+ + E_{D_1\cdots D_{n-1}} 1_{0 < I_{n-1} < s_2^n(D_1\cdots D_{n-1})}[-k_2 + G(Q_2(I_{n-1}); I_{n-1})] + 1_{I_{n-1} > s_2^n(D_1\cdots D_{n-1})} G(0; I_{n-1}) + 1_{I_{n-1}=0}[\Delta_2^n(D_1\cdots D_{n-1}) - k_2]^+.$$

Some calculations will lead to

$$\frac{\partial H_1(Q_1)}{\partial Q_1} = (c_f - c_s) - h\sum_{i=1}^{n+L_2-1} E_{D_1\ldots D_{n+L_2-1}}\Big\{1_{Q_1 - \sum_{j=1}^{i} D_j > 0} - E_{D_1\cdots D_{n-1}} E_{D_n\cdots D_N | D_1\cdots D_{n-1}} 1_{0 \le I_{n-1} < s_2^n(D_1\cdots D_{n-1})} 1_{Q_1 > Y_{1,n+L_2-1}} \left[h\sum_{k=n+L_2}^{N-1} 1_{Q_1+Q_2-Y_{1k}>0} + (c_f - v)1_{Q_1+Q_2-Y_{1N}>0}\right] + 1_{I_{n-1} \ge s_2^n(D_1\cdots D_{n-1})}\left[h\sum_{k=n+L_2}^{N-1} 1_{Q_1-Y_{1k}>0} + (c_f - v)1_{Q_1-Y_{1N}>0}\right]\Big\}. \quad (A4)$$

In deriving the above, terms from differentiations on the upper and lower limits of integrations cancel each other, where Q_2 is a function of Q_1 solved by Theorem 1. Observing that

$$\left.\frac{\partial H_1(Q_1)}{\partial Q_1}\right|_{Q_1\to 0} = c_f - c_s > 0, \quad \left.\frac{\partial H_1(Q_1)}{\partial Q_1}\right|_{Q_1\to\infty} = -c_s - (N-1)h < 0,$$

there is at least one local maximal point for $H_1(Q_1)$. We suppose the maximal point is $Q_1 = S_1^n$, satisfying $\partial H_1(Q_1)/\partial Q_1|_{Q_1 = S_1^n} = 0$. On the other hand, if $Q_2 = 0$, then the optimal value to solve $\partial H_1(Q_1)/\partial Q_1|_{Q_1=0} = 0$ is S_1. Observing that the left side of (A4) monotonically decreases in both Q_1 and Q_2 if treating Q_2 as independent of Q_1, we can clearly conclude that $S_1^n \le S_1$.

(2) Since $Q_1 = S_1$ obviously provides a sub-optimal solution to Problem PP, we have $\Delta_1^n \ge \Delta_1$.

(3) With the above properties, this result follows.

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